MATH 225 March 28, 2002

Name: key

Quiz 12

Justify all your work. Partial credit will be given if you show your reasoning.

(1) Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $w = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$. Determine if w is in $Col\ A$. Is w in $Nul\ A$?

First note that

$$\det A = 1 \cdot 4 - 2 \cdot 3 = -2.$$

Since $\det A \neq 0$ we know that A has an inverse. Thus, we can solve the matrix equation $A\vec{x} = \vec{w}$ by multiplying A^{-1} on the left of both sides of the equation:

$$\vec{x} = A^{-1}\vec{w}.$$

Therefore, $\vec{w}$ is a vector in $\mathbb{R}^2$ for which $A\vec{x} = \vec{w}$ for some $\vec{x} \in \mathbb{R}^2$, so $\vec{w} \in Col\ A$.

Observe that

$$A\vec{w} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}.$$

Since $A\vec{w} \neq \vec{0}$ we see that, by definition, $\vec{w}$ is not in $Nul\ A$.

(2) Let $A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 & 15 \end{bmatrix}$.

(a) Find k such that $Nul\ A$ is a subspace of $\mathbb{R}^k$.

The null space of a $n \times m$ matrix is a subspace of $\mathbb{R}^m$, and since A is 3×5, $Nul\ A$ is a subspace of $\mathbb{R}^5$, so choose $k = 5$. This is easy to verify from the definition of $Nul\ A$ as

$$Nul\ A = \{\vec{w} \in \mathbb{R}^5\ : A\vec{w} = \vec{0}\}.$$

(b) Find n so that $Col\ A$ is a subspace of $\mathbb{R}^n$.

The column space of a $n \times m$ matrix is a subspace of $\mathbb{R}^n$, so since A is 3×5, it is evident that $n = 3$. This is also easy to verify from the definition of $Col\ A$ as the span of the columns of A, or (equivalently)

$$Col\ A = \{\vec{b} \in \mathbb{R}^3\ :\ A\vec{x} = \vec{b} \text{ for some } \vec{x} \in \mathbb{R}^5\}.$$