MATH 225 April 2, 2002

Name: key

Quiz 13

Justify all your work. Partial credit will be given if you show your reasoning.

Let

$$A = \begin{bmatrix} 1 & 2 & -5 & 11 & -3 \\ 2 & 4 & -5 & 15 & 2 \\ 1 & 2 & 0 & 4 & 5 \\ 3 & 6 & -5 & 19 & -2 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 2 & 0 & 4 & 5 \\ 0 & 0 & 5 & -7 & 8 \\ 0 & 0 & 0 & 0 & -9 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Assume A is row equivalent to B.

(1) Find a basis for $Col\ A$.

Since A is row equivalent to B, we see that the pivot columns of A are columns one, three, and five (from left to right). Thus, a basis for the column space of A is

$$\left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} -5 \\ -5 \\ 0 \\ -5 \end{bmatrix} \begin{bmatrix} -3 \\ 2 \\ 5 \\ -2 \end{bmatrix} \right\}.$$

(2) Find a basis for $Nul\ A$. First we completely row reduce B, yielding

$$B \sim \begin{bmatrix} 1 & 2 & 0 & 4 & 0 \\ 0 & 0 & 1 & -\frac{7}{5} & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Thus, the general solution to the homogeneous equation $A\vec{x} = \vec{0}$ is given by

$$\begin{cases} x_1 = -2x_2 - 4x_4 \\ x_3 = \frac{7}{5}x_4 \\ x_2, x_4 \text{ free} \end{cases}.$$

Therefore, each solution to $A\vec{x} = \vec{0}$ can be given by a choice of x_1, x_2, x_3, x_4 satisfying

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -2x_2 - 4x_4 \\ x_2 \\ \frac{7}{5}x_4 \\ x_4 \end{bmatrix} = x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -4 \\ 0 \\ \frac{7}{5} \\ 1 \end{bmatrix}.$$

Therefore, a basis for $Nul\ A$ is given by

$$\left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ 0 \\ \frac{7}{5} \\ 1 \end{bmatrix} \right\}.$$