MATH 225 April 16, 2002

Name: key *Due April 18, 2002*

Quiz 14

Justify all your work. Partial credit will be given if you show your reasoning.

(1) Let

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & -3 & 0 \\ 4 & -13 & 1 \end{bmatrix}.$$

Find a basis for the eigenspace corresponding to the eigenvalue $\lambda = -2$ of A (i.e., You may assume $\lambda = -2$ is an eigenvalue for A. You need to find the basis for the corresponding eigenspace).

Solution:

(1) Solving the matrix equation $(A + 2I)\vec{x} = \vec{0}$ shows that a basis for the eigenspace corresponding to the eigenvalue $\lambda = -2$ is

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix} \right\}.$$

(1) Let

$$A = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 1 & 0 & 2 & 0 \\ 1 & 0 & 0 & 2 \end{bmatrix}.$$

(2) Find **and solve** the characteristic equation of A.
(3) List the multiplicity of each eigenvalue.
(4) Find a basis for the eigenspace corresponding to each eigenvalue.

Solution

(1) Rewriting the equation $\det(A - \lambda I) = 0$ results in the polynomial

$$(\lambda - 4)^2(\lambda - 2)^2 = 0.$$

(2) From the characteristic polynomial above, we find that $\lambda = 2$ is an eigenvalue of multiplicity 2 and $\lambda = 4$ is an eigenvalue of multiplicity 2.
(3) From Row reducing the augmented matrix corresponding to $(A - 2I)\vec{x} = \vec{0}$, we find a basis for the eigenspace corresponding to $\lambda = 2$ is

$$\left\{ \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}.$$

Row reducing the augmented matrix corresponding to $(A - 4I)\vec{x} = \vec{0}$, we find that a basis for the eigenspace corresponding to $\lambda = 4$ is

$$\left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\}.$$