

Name: key

(Practice) Quiz 2

Justify all your work. Partial credit will be given if you show your reasoning.

1. For what values of h is $\vec{y} = \begin{bmatrix} h \\ -3 \\ -5 \end{bmatrix}$ in the span of the vectors $\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$ and

$$\vec{v}_2 = \begin{bmatrix} -2 \\ 1 \\ 7 \end{bmatrix}$$

Equivalently, for what values of h does the vector equation

$$x_1 \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix} + x_2 \begin{bmatrix} -2 \\ 1 \\ 7 \end{bmatrix} = \begin{bmatrix} h \\ -3 \\ -5 \end{bmatrix}$$

have a solution?

This vector equation is equivalent to the system

$$\begin{cases} x_1 - 2x_2 = h \\ x_2 = -3 \\ -2x_1 + 7x_2 = -5 \end{cases}$$

Plugging x_2 into the first equation we find $x_1 = h - 6$. Then using this information in the third equation we find

$$-2(h - 6) - 21 = -5 \implies h = -2.$$

Coincidentally, with this choice of h the system has the solution $x_1 = -8$, $x_2 = -3$.