

Name: key  
 Quiz 3

*Justify all your work. Partial credit will be given if you show your reasoning.*

1. Let  $A = \begin{bmatrix} 1 & 0 & 5 \\ -2 & 1 & -6 \\ 0 & 2 & 8 \end{bmatrix}$  and  $\vec{b} = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}$ . Is  $\vec{b}$  a linear combination of the vectors formed by the columns of  $A$ ? Equivalently, is  $b$  in the span of the columns of  $A$ ?

*The question asked is if we can find suitable real numbers  $x_1, x_2$  and  $x_3$  such that*

$$x_1 \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} + x_3 \begin{bmatrix} 5 \\ -6 \\ 8 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}.$$

*The solution set to this vector equation is the same as the solution set to the matrix*

*equation  $A\vec{x} = \vec{b}$ , where  $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ . Thus, we need to know whether or not*

*the matrix equation  $A\vec{x} = \vec{b}$  has a solution. To that end we form the associated augmented matrix and begin the row reduction process:*

$$\begin{bmatrix} 1 & 0 & 5 & 2 \\ -2 & 1 & -6 & -1 \\ 0 & 2 & 8 & 6 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 5 & 2 \\ 0 & 1 & 4 & 3 \\ 0 & 2 & 8 & 6 \end{bmatrix} \\ \sim \begin{bmatrix} 1 & 0 & 5 & 2 \\ 0 & 1 & 4 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

*The general solution to the corresponding system is*

$$\begin{cases} x_1 = 2 - 5x_3 \\ x_2 = 3 - 4x_3 \\ x_3 \text{ is free} \end{cases}.$$

*Since the system is consistent, we can find real numbers  $x_1, x_2, x_3$  as desired. Thus,  $\vec{b}$  is a linear combination of the vectors formed by the columns of  $A$ . Equivalently,  $\vec{b}$  is in the span of the columns of  $A$ .*