

Name: key
 Quiz 4

Justify all your work. Partial credit will be given if you show your reasoning.

1. Decide if the vectors

$$\begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} -3 \\ -5 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \\ -6 \end{bmatrix}$$

are linearly independent. Give reasons for your answer.

Let $A = \begin{bmatrix} 1 & -3 & 0 \\ 3 & -5 & 5 \\ -2 & 6 & -6 \end{bmatrix}$ and $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$. The vectors given above are linearly independent if and only if the trivial solution is the only solution to the matrix equation $A\vec{x} = \vec{0}$, where $\vec{0}$ is the zero vector. To determine the solutions to this matrix equation we form the associated augmented matrix and row reduce:

$$\begin{aligned} \left[\begin{array}{cccc} 1 & -3 & 0 & 0 \\ 3 & -5 & 5 & 0 \\ -2 & 6 & -6 & 0 \end{array} \right] &\sim \left[\begin{array}{cccc} 1 & -3 & 0 & 0 \\ 0 & 4 & 5 & 0 \\ 0 & 0 & -6 & 0 \end{array} \right] \\ &\sim \left[\begin{array}{cccc} 1 & -3 & 0 & 0 \\ 0 & 1 & 5/4 & 0 \\ 0 & 0 & -6 & 0 \end{array} \right] \\ &\sim \left[\begin{array}{cccc} 1 & 0 & 15/4 & 0 \\ 0 & 1 & 5/4 & 0 \\ 0 & 0 & -6 & 0 \end{array} \right] \\ &\sim \left[\begin{array}{cccc} 1 & 0 & 15/4 & 0 \\ 0 & 1 & 5/4 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right] \\ &\sim \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right] \end{aligned}$$

From this we can conclude that the associated matrix equation $A\vec{x} = \vec{0}$ has only the trivial solution. So the vectors given above are linearly independent.