

Name: key  
 Quiz 6

*Justify all your work. Partial credit will be given if you show your reasoning.*

1. **Without using the formula**, Find the inverse of  $\begin{bmatrix} 7 & 9 \\ -6 & -8 \end{bmatrix}$  if it exists. What is the formula for the inverse of a  $2 \times 2$  matrix  $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ ?

*Form the appropriate  $2 \times 4$  matrix and begin the row reduction process.*

$$\begin{aligned} \begin{bmatrix} 7 & 9 & 1 & 0 \\ -6 & -8 & 0 & 1 \end{bmatrix} &\sim \begin{bmatrix} 1 & 9/7 & 1/7 & 0 \\ -6 & -8 & 0 & 1 \end{bmatrix} \\ &\sim \begin{bmatrix} 1 & 9/7 & 1/7 & 0 \\ 0 & -2/7 & 6/7 & 1 \end{bmatrix} \\ &\sim \begin{bmatrix} 1 & 9/7 & 1/7 & 0 \\ 0 & 1 & -3 & -7/2 \end{bmatrix} \\ &\sim \begin{bmatrix} 1 & 0 & 4 & 9/2 \\ 0 & 1 & -3 & -7/2 \end{bmatrix} \end{aligned}$$

*Thus,*

$$\begin{bmatrix} 7 & 9 \\ -6 & -8 \end{bmatrix}^{-1} = \begin{bmatrix} 4 & 9/2 \\ -3 & -7/2 \end{bmatrix}.$$

*Note that this agrees with the general formula for the inverse of a  $2 \times 2$  matrix:*

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

2. Suppose that  $(B - C)A = 0$ , where  $B$  and  $C$  are  $m \times n$  matrices and  $A$  is invertible. Show that  $B = C$ . Your answer must be logically correct and persuasive.

*We can apply<sup>1</sup> the distributive property of matrix multiplication / addition to the left-hand side of the above equation to get*

$$BA - CA = 0.$$

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<sup>1</sup>Note that other valid ways to prove this exist. Also note that the given proof is not necessarily the most efficient (i.e., shortest) proof.

*Adding  $CA$  to both sides yields*

$$BA = CA.$$

*Finally, multiplying by  $A^{-1}$  on both sides of this equation we find that*

$$BAA^{-1} = CAA^{-1},$$

*or that*

$$BI = CI,$$

*or that*

$$B = C.$$