

$$\text{① Let } A = \begin{bmatrix} 2 & -5 & 0 \\ -1 & 3 & -4 \\ 6 & -8 & -7 \\ 3 & 0 & 9 \end{bmatrix} \quad B = \begin{bmatrix} 4 & -6 \\ 7 & 1 \\ 3 & 2 \end{bmatrix} \quad C = \begin{bmatrix} 3 & -8 \\ -1 & 3 \end{bmatrix}$$

$$D = \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix} \quad I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad E = \begin{bmatrix} 1 & 5 & 0 \\ -2 & -7 & 6 \\ 1 & 3 & -4 \end{bmatrix}$$

a) Compute AB .

$$AB = \begin{bmatrix} 2 & -5 & 0 \\ -1 & 3 & -4 \\ 6 & -8 & -7 \\ 3 & 0 & 9 \end{bmatrix} \begin{bmatrix} 4 & -6 \\ 7 & 1 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 8 - 35 + 0 & -12 - 5 + 0 \\ -4 + 21 - 12 & 6 + 3 - 8 \\ 24 - 56 - 21 & -36 - 8 - 14 \\ 12 + 0 + 27 & -18 + 0 + 18 \end{bmatrix}$$
$$= \begin{bmatrix} -27 & -17 \\ 5 & 1 \\ -53 & -58 \\ 39 & 0 \end{bmatrix}$$

b) Compute A^T , I_2^T , and C^T .

$$A^T = \begin{bmatrix} 2 & -5 & 6 & 3 \\ -1 & 3 & -8 & 0 \\ 0 & -4 & -7 & 9 \end{bmatrix}$$

$$I_2^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$C^T = \begin{bmatrix} 3 & -1 \\ -8 & 3 \end{bmatrix}$$

c) Compute C^{-1} , D^{-1} .

$$C^{-1} = \frac{1}{9-8} \begin{bmatrix} 3 & 8 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 8 \\ 1 & 3 \end{bmatrix}$$

$$D^{-1} = \frac{1}{0+2} \begin{bmatrix} 0 & -2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

d) Compute CD and $(D^{-1})(C^{-1})$

$$CD = \begin{bmatrix} 3 & -8 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 11 & 6 \\ -4 & -2 \end{bmatrix}$$

$$D^{-1}C^{-1} = \begin{bmatrix} 0 & -1 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 3 & 8 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} -1 & -3 \\ 2 & \frac{11}{2} \end{bmatrix}$$

e) Compute $(CD)^{-1}$.

$$\begin{aligned} (CD)^{-1} &= \begin{bmatrix} 11 & 6 \\ -4 & -2 \end{bmatrix}^{-1} = \frac{1}{-22+24} \begin{bmatrix} -2 & -6 \\ 4 & 11 \end{bmatrix} \\ &= \begin{bmatrix} -1 & -3 \\ 2 & \frac{11}{2} \end{bmatrix} \end{aligned}$$

f) What do you notice on this page?

$$(CD)^{-1} = D^{-1}C^{-1}$$

g) Compute $(C^{-1})^{-1}$. What do you notice?

$$(C^{-1})^{-1} = \begin{bmatrix} 3 & 8 \\ 1 & 3 \end{bmatrix}^{-1} = \frac{1}{9-8} \begin{bmatrix} 3 & -8 \\ -1 & 3 \end{bmatrix}$$

$$(C^{-1})^{-1} = C = \begin{bmatrix} 3 & -8 \\ -1 & 3 \end{bmatrix}$$

h) Compute $(D^T)^{-1}$ and $(D^{-1})^T$. What do you notice?

$$(D^T)^{-1} = \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}^{-1} = \frac{1}{0+2} \begin{bmatrix} 0 & 1 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1/2 \\ -1 & 1/2 \end{bmatrix}$$

$$(D^{-1})^T = \begin{bmatrix} 0 & -1 \\ 1/2 & 1/2 \end{bmatrix}^T = \begin{bmatrix} 0 & 1/2 \\ -1 & 1/2 \end{bmatrix}, \quad (D^T)^{-1} = (D^{-1})^T$$

i) Use inverses to solve: $3x_1 - 8x_2 = 5$
 $-x_1 + 3x_2 = 6$

The system is equivalent to

$$\begin{bmatrix} 3 & -8 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$$

\Leftrightarrow

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 & 8 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 63 \\ 23 \end{bmatrix}$$

k) Find CI_2 and I_2D . What do you notice

$$CI_2 = \begin{bmatrix} 3 & -8 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -8 \\ -1 & 3 \end{bmatrix}$$

$$I_2D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}$$

l) Find $(A^T)^T$. What do you notice

$$(A^T)^T = \begin{bmatrix} 2 & -1 & 6 & 3 \\ -5 & 3 & -8 & 0 \\ 0 & -4 & -7 & 9 \end{bmatrix}^T = \begin{bmatrix} 2 & -5 & 0 \\ -1 & 3 & -4 \\ 6 & -8 & -7 \\ 3 & 0 & 9 \end{bmatrix}$$

$$(A^T)^T = A$$

m) Find $(CD)^T$ and D^TC^T . What do you notice.

$$(CD)^T = \begin{bmatrix} 11 & 6 \\ -4 & -2 \end{bmatrix}^T = \begin{bmatrix} 11 & -4 \\ 6 & -2 \end{bmatrix}$$

$$D^TC^T = \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 3 & -1 \\ -8 & 3 \end{bmatrix} = \begin{bmatrix} 11 & -4 \\ 6 & -2 \end{bmatrix}$$

$$(CD)^T = D^TC^T$$