MATH 225 March 7, 2002

Name: ____________________________

Quiz 9

Justify all your work. Partial credit will be given if you show your reasoning.

(1) Find the determinant of

$$A = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 7 & -1 & 0 & 0 \\ 2 & 6 & 3 & 0 \\ 5 & -8 & 4 & -3 \end{bmatrix}$$

by using the cofactor expansion of A along the first row: i.e.,

$$\begin{vmatrix} 4 & 0 & 0 & 0 \\ 7 & -1 & 0 & 0 \\ 2 & 6 & 3 & 0 \\ 5 & -8 & 4 & -3 \end{vmatrix} = 4 \cdot (-1)^{1+1} \cdot \begin{vmatrix} -1 & 0 & 0 \\ 6 & 3 & 0 \\ -8 & 4 & -3 \end{vmatrix} + 0 \cdot (-1)^{1+2} \cdot \begin{vmatrix} 7 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 4 & -3 \end{vmatrix}$$

$$+0 \cdot (-1)^{1+3} \cdot \begin{vmatrix} 7 & -1 & 0 \\ 2 & 6 & 0 \\ 5 & -8 & -3 \end{vmatrix} + 0 \cdot (-1)^{1+4} \cdot \begin{vmatrix} 7 & -1 & 0 \\ 2 & 6 & 3 \\ 5 & -8 & 4 \end{vmatrix}.$$

Solution:

Since each term above except the first is zero we only need to compute the determinant for the first term. To do this we can either expand along the first row or use the fact that the determinant of a diagonal matrix is the product of the entries on the main diagonal. Thus, using the latter method,

$$\begin{vmatrix} -1 & 0 & 0 \\ 6 & 3 & 0 \\ -8 & 4 & -3 \end{vmatrix} = (-1)(3)(-3) = 9.$$

Hence,

$$\begin{aligned} \begin{vmatrix} 4 & 0 & 0 & 0 \\ 7 & -1 & 0 & 0 \\ 2 & 6 & 3 & 0 \\ 5 & -8 & 4 & -3 \end{vmatrix} &= 4 \cdot (-1)^{1+1} \cdot \begin{vmatrix} -1 & 0 & 0 \\ 6 & 3 & 0 \\ -8 & 4 & -3 \end{vmatrix} + 0 \cdot (-1)^{1+2} \cdot \begin{vmatrix} 7 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 4 & -3 \end{vmatrix} \\ &\quad +0 \cdot (-1)^{1+3} \cdot \begin{vmatrix} 7 & -1 & 0 \\ 2 & 6 & 0 \\ 5 & -8 & -3 \end{vmatrix} + 0 \cdot (-1)^{1+4} \cdot \begin{vmatrix} 7 & -1 & 0 \\ 2 & 6 & 3 \\ 5 & -8 & 4 \end{vmatrix} \\ &= 4 \cdot 1 \cdot 9 \\ &= 36. \end{aligned}$$

Expanding along the first row would yield the same answer.