

- Recall the definition of the determinant via cofactor expansion.
- If  $A$  is a triangular matrix, then  $\det A$  is the product of the entries along the main diagonal of  $A$ .
- Let  $A$  be a square matrix.
  - If a multiple of one row of  $A$  is added to another row to obtain the matrix  $B$ , then  $\det A = \det B$ .
  - If Two rows of  $A$  are interchanged to produce the matrix  $B$ , then  $\det B = -\det A$ .
  - If one row of  $A$  is multiplied by  $K$  to produce  $B$ , then  $\det B = k \det A$ .
- A square matrix  $A$  is invertible if and only if  $\det A \neq 0$ .
- If  $A$  is an  $n \times n$  matrix, then  $\det A^T = \det A$ .
- If  $A$  and  $B$  are  $n \times n$  matrices, then  $\det(AB) = \det(A) \det(B)$ .
- If  $A$  is a  $2 \times 2$  matrix, the area of the parallelogram determined by the columns of  $A$  is  $|\det A|$ . If  $A$  is a  $3 \times 3$  matrix, the volume of the parallelepiped determined by the columns of  $A$  is  $|\det A|$ .

**Special Note:** The area of a triangle formed by connecting the points defined by  $\vec{0}$ ,  $\vec{v}_1$ , and  $\vec{v}_2$  in  $\mathbb{R}^2$  is half the area of the parallelogram determined by  $\vec{v}_1, \vec{v}_2$ , i.e.,

$$\frac{1}{2} |\det A|,$$

where  $A$  is the matrix with columns consisting of  $\vec{v}_1$  and  $\vec{v}_2$ .