MATH 234: HOMEWORK 1

SOLUTIONS

1. Find the derivative of $f(x) = x^{5/2}$.

Solution. By the Power Rule

$$f'(x) = (5/2)x^{3/2}.$$

2. Find the derivative of $f(x) = 2\sqrt{x}$ at $x = 4$.

Solution. Well $2\sqrt{x} = 2x^{1/2}$. So by the Power Rule

$$f'(x) = x^{-1/2}.$$

So $f'(4) = 4^{-1/2} = 1/2$.

3. Find the equation of the line which is tangent to the curve $y = \sqrt{x}$ at $x = 9$.

Solution. Well $\sqrt{x} = x^{1/2}$. So by the Power Rule

$$f'(x) = 1/2x^{-1/2}.$$

So

$$f'(9) = \frac{9^{-1/2}}{2} = \frac{1}{6}.$$

Now write

$$\begin{aligned} f(9) &= (1/6)9 + b \\ 3 &= 3/2 + b \\ 3 - 3/2 &= b \\ 3/2 &= b \end{aligned}$$

Thus the equation for the tangent line is $y = (1/6)x + (3/2)$.

4. Determine whether or not the following limit exists. If so, compute the limit.

$$\lim_{x \to -1} \frac{x^2 - 1}{x + 1}$$

Solution. For $x \neq -1$ we have

$$\frac{x^2 - 1}{x + 1} = \frac{(x + 1)(x - 1)}{(x + 1)} = (x - 1),$$

which is a continuous function. Thus

$$\lim_{x\to -1} \frac{x^2-1}{x+1} = \lim_{x\to -1}(x-1) = -2.$$

5. Determine whether or not the following limit exists. If so, compute the limit.

$$\lim_{x\to 3} \frac{\sqrt{x}-4}{x^3+27}$$

Solution. Since our expression,

$$\frac{\sqrt{x}-4}{x^3+27},$$

is continuous at 3, we can evaluate it to see

$$\lim_{x\to 3} \frac{\sqrt{x}-4}{x^3+27} = \frac{\sqrt{3}-4}{3^3+27} = \frac{\sqrt{3}-4}{54}.$$

6. If $f(x) = 1/x^{2/3}$, then what is:

$$\lim_{h\to 0} \frac{f(-8+h)-f(-8)}{h}$$

Solution. Well

$$\lim_{h\to 0} \frac{f(-8+h)-f(-8)}{h} = f'(-8).$$

Since $f'(x) = (-2/3)x^{-5/3}$,

$$f'(-8) = \frac{-2}{3\cdot((-8)^{1/3})^5} = \frac{1}{3\cdot 2^4} = \frac{1}{48}.$$

7. Compute the limit:

$$\lim_{x\to\infty} \frac{2x^2+1}{x^2+1}$$

Solution. Well for $x \neq 0$,

$$\frac{2x^2+1}{x^2+1} = \frac{2+1/x^2}{1+1/x^2},$$

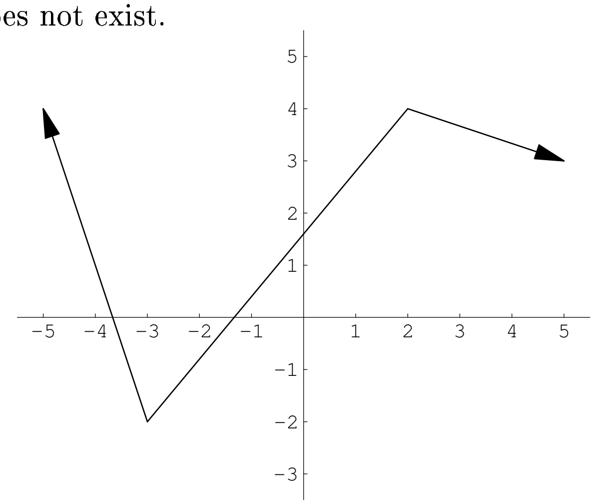
and we see that

$$\lim_{x\to\infty} \frac{2+1/x^2}{1+1/x^2} = 2.$$

Hence

$$\lim_{x\to\infty} \frac{2x^2+1}{x^2+1} = 2.$$

8. For the graphed function below, state the x-values for which the derivative does not exist.



Solution. Well the "kinks" are at $x = -3$ and $x = 2$. That's were the function does not have a derivative.

9. Which of the following properties are satisfied by the following function:

$$f(x) = \begin{cases} x^2 + 1 & \text{for } x < 0 \\ 1 & \text{for } x = 0 \\ 5x + 1 & \text{for } x > 0 \end{cases}$$

(I) $f(x)$ is continuous.
(II) $f(x)$ is differentiable for all x.
(III) $f(x)$ is differentiable at $x = -2$.

Solution. We see that (I) is true as f is defined to be 1 at $x = 0$, while

$$\lim_{x \to 0} x^2 + 1 = 1 \quad \text{and} \quad \lim_{x \to 0} 5x + 1 = 1.$$

(II) is false as $\dfrac{d}{dx}(x^2 + 1)$ is 0 at 0 and $\dfrac{d}{dx}(5x + 1)$ is 5 at 0.
(III) is true as $x^2 + 1$ is continuous at $x = -2$.

10. A company is planning to manufacture a new blender. After conducting extensive market surveys, the research department estimates a weekly demand of 600 blenders at a price of \$50 per blender and a weekly demand of 800 blenders at a price of \$40 per blender. Assuming the demand equation is linear, use the research department's estimates to find the revenue $R(x)$ in terms of the demand x.

Solution. Since we are assuming the demand with respect to price is linear, we see that the price with respect to demand must also be linear. The slope of our price function, call it $P(x)$, is

$$\frac{40-50}{800-600}=\frac{-1}{20}.$$

Solving the equation $40=-800/20+b$ for b we find $b=80$ and so,

$$P(x)=-x/20+80.$$

Since revenue is price times demand, the formula for $R(x)$ is

$$R(x)=-x^2/20+80x.$$