MATH 234: HOMEWORK 10

SOLUTIONS

1. Let $F(x,y,z) = \dfrac{xz}{y^2z+x} - 5x^2y^3 + 2y$. Compute $\dfrac{\partial}{\partial y}F(x,y,z)$ and evaluate it at the point $(-1,0,1)$.

Solution. Write

$$\begin{aligned}\frac{\partial}{\partial y}F(x,y,z) &= \frac{\partial}{\partial y}\left((xz)(y^2z+x)^{-1} - 5x^2y^3 + 2y\right),\\ &= (xz)(-1)(y^2z+x)^{-2}(2yz) - 15x^2y^2 + 2.\end{aligned}$$

Evaluating this at $(-1,0,1)$ you get 2.

2. Let $P(x,y,z) = xy + 2x^3\sqrt{y^2-1}$. Compute $\dfrac{\partial}{\partial z}P(x,y,z)$ and evaluate it at the point $(2,1,3)$.

Solution. Write

$$\frac{\partial}{\partial z}P(x,y,z) = 0.$$

Done!

3. Let $f(x,y) = x^2 + 2xy + 5y^2 + 2x + 10y - 3$. At which point does $f(x,y)$ have a *possible* maximum or minimum?

Solution. Look at

$$\frac{\partial}{\partial x}f(x,y) = 2x + 2y + 2,$$

and

$$\frac{\partial}{\partial y}f(x,y) = 2x + 10y + 10.$$

Setting these both equal to zero and subtracting the first equation from the second we get

$$8y + 8 = 0.$$

Hence $y = -1$ and $x = 0$.

4. Let $f(x,y) = x^4 - y^2 - 2x^2 + 2y - 7$. The first partial derivatives of $f(x,y)$ are zero at the points $(0,1)$ and $(-1,1)$. Use the second-derivative test to determine the nature of $f(x,y)$ at each of these points.

Solution. Write

$$\frac{\partial}{\partial x} f(x,y) = 4x^3 - 4x, \qquad \frac{\partial}{\partial y} f(x,y) = -2y + 2,$$

$$\frac{\partial^2}{\partial x^2} f(x,y) = 12x^2 - 4, \qquad \frac{\partial^2}{\partial y^2} f(x,y) = -2,$$

$$\frac{\partial^2}{\partial x \partial y} f(x,y) = 0.$$

Now

$$D(0,1) = (12 \cdot 0^2 - 4)(-2) - 0^2 = 8 > 0,$$

and

$$\frac{\partial^2}{\partial y^2} f(x,y) = -2 < 0,$$

so $(0,1)$ is a local max. Also

$$D(-1,1) = (12 \cdot (-1)^2 - 4)(-2) - 0^2 = -16 < 0,$$

so $(-1,1)$ is neither a local max nor is it a local min.

5. What values of x and y maximize the function $g(x,y) = x + 3y$ subject to the constraint $x^2 + 9y^2 = 72$?

Solution. Let's use Lagrange's method. Set

$$F(x,y,\lambda) = x + 3y + \lambda(x^2 + 9y^2 - 72).$$

So

(1) $$\frac{\partial}{\partial x} F(x,y,\lambda) = 1 + 2x\lambda = 0,$$

(2) $$\frac{\partial}{\partial y} F(x,y,\lambda) = 3 + 18y\lambda = 0,$$

(3) $$\frac{\partial}{\partial \lambda} F(x,y,\lambda) = x^2 + 9y^2 - 72 = 0.$$

Solving equations (1) and (2) for λ we find

$$\frac{-1}{2x} = \lambda = \frac{-1}{6y}.$$

So $2x = 6y$ and we see that $x = 3y$. Substituting this into (3) we get

$$(3y)^2 + 9y^2 - 72 = 0.$$

So $18y^2 - 72 = 0$. Hence $y = 2$ and $x = 6$. Note we discard the negative answers as they are clearly less maximal than the positive solutions.