MATH 234: HOMEWORK 2

SOLUTIONS

1. Find the derivative with respect to x of $f(x) = -\dfrac{2}{3x^4}$.

Solution. Write:

$$\frac{d}{dx}\left(-\frac{2}{3x^4}\right) = \frac{d}{dx}(-(2/3)x^{-4}) = (8/3)x^{-5}$$

2. Find the derivative with respect to x of $g(x) = \dfrac{2x - (x^2+1)^7}{3}$.

Solution. Write:

$$\begin{aligned}\frac{d}{dx}\left(\frac{2x-(x^2+1)^7}{3}\right) &= \frac{d}{dx}((2/3)x - (1/3)(x^2+1)^7) \\ &= 2/3 - (7/3)(x^2+1)^6(2x)\end{aligned}$$

3. Find the derivative with respect to x of $h(x) = \dfrac{5}{x^3 - 4x^2 + 2}$.

Solution. Write:

$$\begin{aligned}\frac{d}{dx}\left(\frac{5}{x^3-4x^2+2}\right) &= \frac{d}{dx}(5(x^3-4x^2+2)^{-1}) \\ &= -5(x^3-4x^2+2)^{-2}(3x^2-8x)\end{aligned}$$

4. Compute:

$$\frac{d}{du}\sqrt{u^4-7}$$

Solution. Write:

$$\frac{d}{du}\sqrt{u^4-7} = \frac{d}{du}(u^4-7)^{1/2} = (1/2)(u^4-7)^{-1/2}(4u^3)$$

5. Compute:

$$\frac{d}{ds}(p^2s^4 - q^4r^3s^2)$$

Solution. Write:

$$\frac{d}{ds}(p^2s^4 - q^4r^3s^2) = 4p^2s^3 - 2q^4r^3s$$

6. Find the second derivative with respect to x of $f(x) = 3x^2 + 6x - 4$.

Solution. Well $f'(x) = 6x + 6$ and so $f''(x) = 6$.

7. Compute:

$$\frac{d}{dx}(-4 + 3\sqrt{x})^4\Big|_{x=4}$$

Solution. Well

$$\frac{d}{dx}(-4 + 3\sqrt{x})^4 = 4(-4 + 3x^{1/2})^3(3/2)(x^{-1/2}),$$

and

$$4(-4 + 3(4^{1/2}))^3(3/2)(4^{-1/2}) = 24.$$

8. Which of the following is the best description of $f'(t)$?

(A) It is approximately equal to $\dfrac{f(t+h) - f(t)}{h}$, as t gets very small.
(B) It is a function which gives the slope of the secant line through any two points.
(C) $f'(t)$ measures the rate of change of $f(t)$ per unit change in t.
(D) $f'(t) = \dfrac{f(t)}{t}$.
(E) When considered as a function, the derivative is the best approximation of the tangent line of $f(x)$.

Solution. (C) is best.

9. On February 1st, a flu epidemic hits the University. The number of people sick at time t (measured in days) is given by the function $P(t)$. The rate at which the epidemic is spreading on February 3 is 110 newly infected people per day. How is the information in the immediately preceding sentence best represented mathematically?

(A) $P'(3) = 110$.
(B) $\dfrac{d}{dt}P(t)\Big|_{t=110}$.
(C) $P(3) = 110$.
(D) None of the above.

Solution. (A) is best.

10. At time $t = 0$, a seed is planted. After t weeks, the height of the plant is given by $f(t) = 0.3t^2 + 0.6t + 0.5$ inches. At what rate is the plant growing after 8 weeks?

Solution. Well, $f'(t) = 0.6t + 0.6$ and $0.6 \cdot 8 + 0.6 = 5.4$ so the answer is 5.4 inches per week.