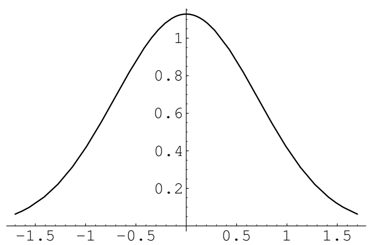
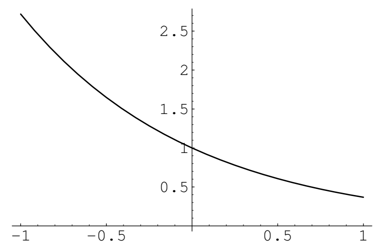
MATH 234: HOMEWORK 3

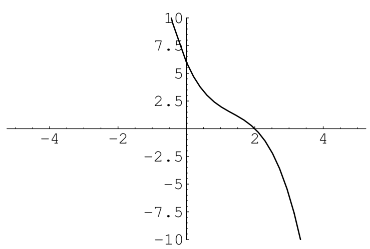
SOLUTIONS

1. Which of the following graphs could represent a function with all of the three following properties?

- $f(x) > 0$ for $x < 0$.
- $f'(x) \leq 0$ for all x.
- $f'(0) = 0$.

(A) 

(B) 

(C) 

(D) None of the above.

Solution. (A) cannot be the graph of $f(x)$ as the graph of (A) is increasing left of the origin. (B) cannot be the graph of $f(x)$ as the graph does not have zero slope at the origin. (C) cannot be the graph of $f(x)$ as it also does not have zero slope at the origin. Hence (D) is the best answer.

2. Where is the function $f(x) = \dfrac{5}{(2x-4)^3}$ increasing?

Solution. We need to take the derivative to figure this out. Write

$$\frac{d}{dx} f(x) = (-30)(2x-4)^{-4}.$$

We see $f'(x)$ is negative, hence $f(x)$ is decreasing for all x where $f(x)$ is defined. Hence $f(x)$ is not increasing for any x on which it is defined.

3. Find the inflection point(s) of $y = 2x^3 - 3x^2 - 12x + 17$.

Solution. We need to find the points where y'' is zero and then see if the concavity is actually changing there. Write

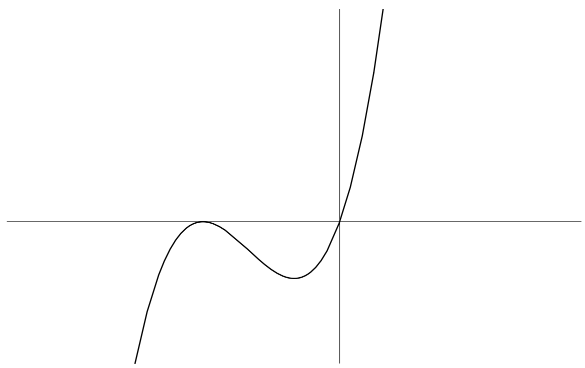
$$\frac{d}{dx}y = 6x^2 - 6x - 12,$$

and

$$\frac{d^2}{dx^2}y = 12x - 6.$$

Hence $y'' = 0$ when $x = 1/2$. Also since $y'' > 0$ when $x > 1/2$ and $y'' < 0$ when $x < 1/2$ we see that $(x, y) = (1/2, 21/2)$ is the only inflection point of $y = 2x^3 - 3x^2 - 12x + 17$.

4. Suppose $f(x)$ is graphed below. Which of the following could be $f(x)$?



(A) $f(x) = 1/x + x^2 + 3x$.
(B) $f(x) = -x^2 + 2x + 5$.
(C) $f(x) = x^2 + 3x$.
(D) $f(x) = x^3 + 5$.
(E) $f(x) = x^3 + 6x^2 + 9x$.

Solution. I think it is pretty clear that $f(x)$ is not (A), (B), or (C), as if $f(x)$ is a polynomial, then it is at least a third degree polynomial. Since (D) touches the x-axis exactly once, (E) must be the correct answer.

5. A manufacture estimates that the profit from producing x units of a commodity is $-x^2 + 40x - 100$ dollars per week. What is the maximum profit he can realize in one week?

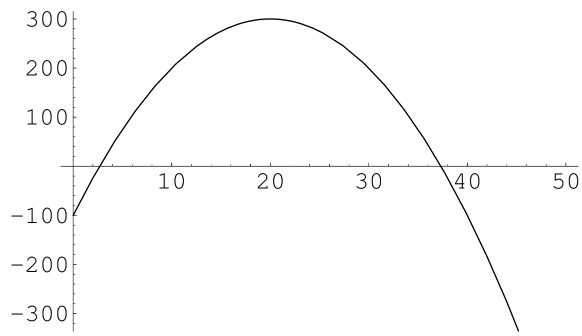
Solution. Since we are looking to maximize profit, let's sketch $-x^2 + 40x - 100$. Write

$$\frac{d}{dx}\left(-x^2 + 40x - 100\right) = -2x + 40.$$

This is 0 when $x = 20$. This is the sole place where the first derivative is zero. Now look at:

$$\frac{d^2}{dx^2}\left(-x^2 + 40x - 100\right) = -2.$$

Thus our curve is concave-down. Here is a sketch of the curve:



So $x = 20$ gives a maximum profit of \$300 per week.

6. Suppose a ball is thrown into the air and after t seconds has a height of $h(t) = -16t^2 + 80t$ feet. When will it reach its maximum height?

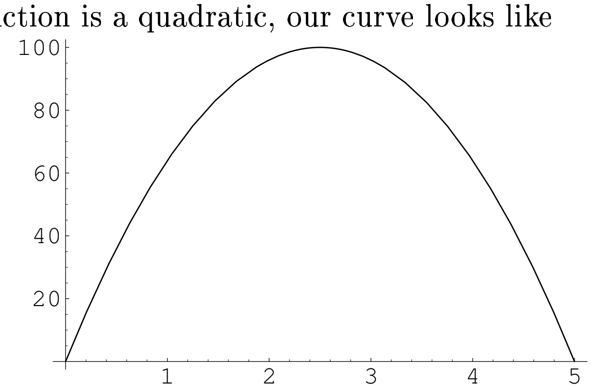
Solution. Again we are maximizing, so let's sketch the curve. Write

$$\frac{d}{dt}h(t) = -32t + 80.$$

This is 0 when $t = 2.5$. Is this a maximum? Well

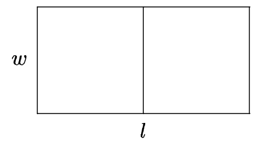
$$\frac{d^2}{dt^2}h(t) = -32.$$

Since our function is a quadratic, our curve looks like



and hence, $t = 2.5$ seconds gives a maximum height.

7. A rectangular corral with total area of 60 square meters is to be fenced off and then divided into 2 rectangular sections by a fence down the middle.



The fencing for the outside costs \$9 per running meter, whereas that for the interior dividing fence costs \$12 per running meter. Which of the following statements hold, if the cost C of the fencing is to be maximized?

(A) The constraint equation is $3w + 2l = 60$.
(B) The objective equation is $2l \cdot w = 60$.
(C) The constraint equation is $w \cdot l = 60$.
(D) The objective equation is $C = 30w + 18l$.
(E) The constraint equation is $C = 12w + 9wl$.
(F) The objective equation is $C = 60 - lw$.

Solution. Let's just write it out. The objective equation is

$$C = 9w + 9w + 9l + 9l + 12w = 30w + 18l. \tag{D}$$

Now the constraint equation is

$$60 = w \cdot l. \tag{C}$$

That does it.

8. A homebuilder's advertisement promises a house with a finished recreation room of 300 square feet. Two perpendicular walls of the room are to be paneled at a cost of \$5 per running foot. A third side will be built out of windows at a cost of \$10 per running foot. The fourth side will use the existing cinder block. What dimensions should the room have to minimize the homebuilder's cost?

Solution. So if l represents the length of one set of walls, and w is the width of the other set, the objective equation is

$$C = 5l + 5w + 10w = 5l + 15w.$$

And the constraint equation is

$$l \cdot w = 300.$$

From this we see that $l = 300/w$. So now we have

$$C = 5(300/w) + 15w = 1500/w + 15w$$

Since we a minimizing, let's sketch the curve. Write

$$\frac{d}{dw}C = -1500/w^2 + 15.$$

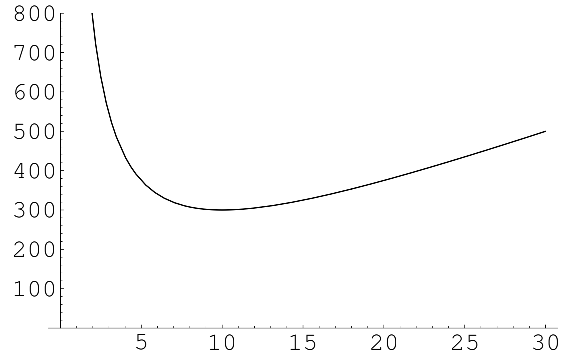
Now, $-1500/w^2 + 15 = 0$ when

$$w^2 - 100 = 0.$$

hence $w = 10$ may be a minimum. To make sure it is, note that

$$\frac{d^2}{ds^2}C = 3000/w^3$$

which is positive for $w = 10$. Here is a sketch of the curve:



Since our initial function was a hyperbola which is concave-up in the relevant domain, we see that $w = 10$ and $l = 30$ feet (from the constraint equation) minimizes the cost.

9. A sports retailer expects to sell 120 sweat suits at a steady rate over the course of the coming year. The cost of placing an order with the wholesaler is \$40. The annual storage cost per sweat suit is \$6 based on the average inventory level. Determine the order size that minimizes ordering and storage expenses.

Solution. If r is the number of orders made and s is the size of each order, the equation for the expenses will be

$$E = 40r + 6(s/2)$$

which is the objective equation. Meanwhile, the constraint equation is

$$r \cdot s = 120.$$

Hence we may rewrite E as

$$E = 40(120/s) + 6(s/2) = 4800/s + 3s.$$

Now since we are looking to minimize E, let's sketch the curve. Write

$$\frac{d}{ds}E = -4800s^{-2} + 3.$$

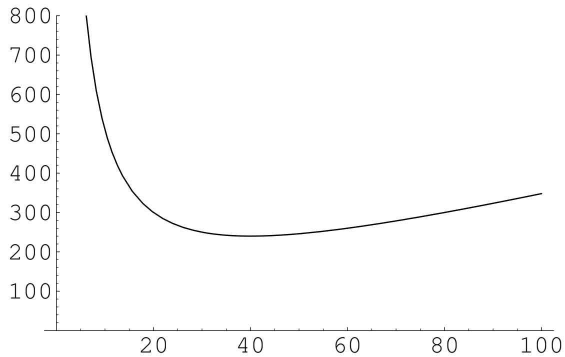
Multiplying both sides by $s^2/3$, we see that this is zero when

$$s^2 - 1600 = 0.$$

Hence $s = 40$ may minimize the expenses. To check, look at

$$\frac{d^2}{ds^2}E = 9600/s^3$$

which is positive for $s = 40$. So we have:



Since our initial function was a hyperbola which is concave-up in the relevant domain, we see that 40 sweat suits per order minimizes expenses.

10. A health food store stocks bottles of multivitamins. It orders equal quantities of stock from its wholesaler at equally spaced points throughout the year. The cost of replacing each order is \$250. Moreover, the cost of keeping a jar of vitamins in inventory is \$1 per year. The store predicts that it will sell 12,500 bottles of vitamins in the next year. How many orders of how many bottles each will result in a minimum cost to the health food store?

Solution. Let r be the number of orders and s be the size of each order. Now our objective equation is

$$C = 250r + (s/2).$$

And our constraint equation is

$$r \cdot s = 12500$$

Hence

$$C = 250(12500/s) + s/2 = 3125000/s + s/2.$$

Now since we are minimizing, we'll sketch the curve. Take the derivative and you get

$$\frac{d}{ds}C = -3125000/s^2 + 1/2.$$

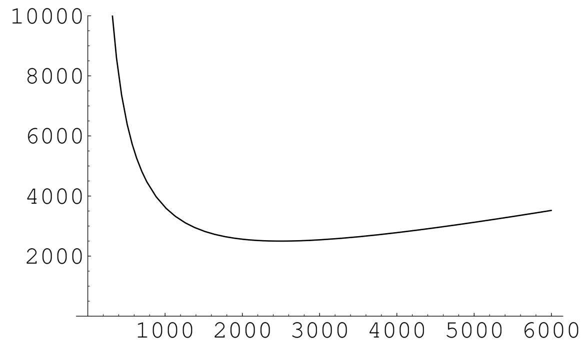
Now $C' = 0$ when

$$s^2 - 6250000 = 0$$

(We multiplied both sides by $2s^2$.) Hence $s = 2500$. Is this a minimum? Well

$$\frac{d^2}{ds^2}C = 6250000/s^3$$

which is positive when $s = 2500$. Now



Shows us that $s = 2500$ is the minimum as our initial function is concave-up in the relevant domain. Using the constraint equation, we see that 5 orders of 2500 bottles each minimizes cost.