MATH 234: HOMEWORK 6

SOLUTIONS

1. If $e^{-x} = 6$, find x in terms of the natural logarithm.

Solution. Well, by the definition of the natural logarithm, we see that

$$-x = \ln(6).$$

Hence $x = -\ln(6)$.

2. Differentiate $e^{x^2} + 2\ln(x^e)$ with respect to x.

Solution. Write

$$\begin{aligned} \frac{d}{dx}\left(e^{x^2} + 2\ln(x^e)\right) &= e^{x^2}(2x) + 2\frac{ex^{e-1}}{x^e} \\ &= e^{x^2}(2x) + 2e/x. \end{aligned}$$

3. Differentiate $x^3\ln(x)$ with respect to x.

Solution. Write

$$\frac{d}{dx}\left(x^3\ln(x)\right) = 3x^2\ln(x) + x^3/x = 3x^2\ln(x) + x^2.$$

4. Use logarithmic differentiation to differentiate

$$4^x \cdot 5^x \cdot 6x^3$$

with respect to x.

Solution. First we'll compute

$$\frac{d}{dx}\ln(4^x \cdot 5^x \cdot 6x^3).$$

Note that

$$\ln(4^x \cdot 5^x \cdot 6x^3) = x\ln(4) + x\ln(5) + \ln(6) + 3\ln(x).$$

So

$$\frac{d}{dx}\ln(4^x \cdot 5^x \cdot 6x^3) = \ln(4) + \ln(5) + 3/x.$$

Hence

$$\frac{d}{dx}\left(4^x \cdot 5^x \cdot 6x^3\right) = 4^x \cdot 5^x \cdot 6x^3(\ln(4) + \ln(5) + 3/x).$$

5. A bacterial culture grows exponentially, that is, $P(t) = 100e^{kt}$, where $P(t)$ is the size of the culture at time t hours. Suppose that after 2 hours the size of the culture is 400. What is k (approximately)?

Solution. Well this is just asking us to solve

$$400 = 100e^{k2}$$

for k. So we want to know when

$$4 = e^{k2}.$$

Write

$$\begin{aligned} 4 &= e^{k2}, \\ \ln(4) &= k2, \\ \ln(4)/2 &= k. \end{aligned}$$

So $k = \ln(4)/2 \approx 0.69$.

6. A radioactive substance is observed to disintegrate at a rate such that $\frac{9}{10}$ of the original amount remains after one year. What is the half-life of this substance?

Solution. If $A(t)$ is the amount of the radioactive substance after t years, then we know from class that

$$A(t) = e^{kt}.$$

Since we know

$$9/10 = e^{k}$$

We see that $k = \ln(9/10)$. Now we want to solve for t where

$$1/2 = e^{\ln(9/10)t}.$$

Taking the natural log of both sides and dividing by $\ln(9/10)$ we find that

$$t = \frac{\ln(1/2)}{\ln(9/10)} \approx 6.579 \text{ years.}$$

7. \$1000 is invested at 6% interest compounded continuously. What is the value of the investment after 5 years?

Solution. So we know $A = Pe^{rt}$. Here $P = 1000$, $r = 0.06$ and $t = 5$. Plug and chug!

$$A = 1000e^{0.06 \cdot 5} \approx 1349.86 \text{ dollars.}$$

8. How much money has to be invested now at 8% continuous interest in order to have \$1000 after 5 years?

Solution. There's more than one way to do this, but I'll simply use the formula we used in the last problem. Now we have

$$1000 = Pe^{0.08\cdot 5}.$$

Solve for P to see that $P = 670.32$ dollars.

9. Determine the percentage rate of change of $f(x) = e^{0.9x}$ at $x = 15$ and $x = 30$.

Solution. Percentage rate of change of $f(x)$ is given by

$$\frac{f'(x)}{f(x)}.$$

Since $f'(x) = 0.9e^{0.9x}$,

$$\frac{f'(x)}{f(x)} = \frac{0.9e^{0.9x}}{e^{0.9x}} = 0.9.$$

Since our final answer is independent of x, the percentage rate of change of $f(x)$ is 90% at both $x = 15$ and $x = 30$.

10. For the demand function $q = 150(245 - p^2)$, find $E(p)$ and determine if the demand is elastic or inelastic (or neither) at the price $p = 7$.

Solution. Well,

$$E(p) = \frac{-pq'(p)}{q(p)}$$

and $q'(p) = -150 \cdot 2 \cdot p$. Hence

$$E(p) = \frac{-p \cdot (-150) \cdot 2 \cdot p}{150(245 - p^2)} = \frac{2p^2}{245 - p^2}.$$

Since this is less than 1 at $p = 7$, then demand is inelastic at $p = 7$.