MATH 234: HOMEWORK 7

SOLUTIONS

1. Suppose that $f(x)$ is an antiderivative of $\dfrac{2}{\sqrt{x}}$ and that $f(0) = 1$. What is $f(9)$?

Solution. So

$$f(x) = \int \frac{2}{\sqrt{x}}\,dx = 4x^{1/2} + C$$

for some constant C. Since $f(0) = 1$ we may solve for C and see that

$$f(x) = 4x^{1/2} + 1.$$

Hence $f(9) = 4 \cdot 9^{1/2} + 1 = 13$.

2. Compute:

$$\int \frac{1}{(x+2)^2}\,dx$$

Solution.

$$\int \frac{1}{(x+2)^2}\,dx = \frac{-1}{(x+2)} + C$$

for any constant C.

3. Compute:

$$\int \left(\frac{x^2}{4} - 4\right)\,dx$$

Solution.

$$\int \left(\frac{x^2}{4} - 4\right)\,dx = \frac{x^3}{12} - 4x + C$$

for any constant C.

4. Compute:

$$\int_0^1 \frac{2}{3-2x}\,dx$$

Solution.

$$\begin{aligned}\int_0^1 \frac{2}{3-2x}\,dx &= \Big[-\ln|3-2x|\Big]_0^1, \\ &= -\ln(3-2) - (-\ln(3-0)).\end{aligned}$$

While it is fine to stop here on an exam, I will continue on:

$$\begin{aligned} -\ln(3-2) - (-\ln(3-0)) &= -\ln(1) + \ln(3) \\ &= \ln(3). \end{aligned}$$

5. Compute:

$$\int_{-1}^{1} e^{-2x}\,dx$$

Solution.

$$\begin{aligned} \int_{-1}^{1} e^{-2x}\,dx &= \Big[(-1/2)e^{-2x}\Big]_{-1}^{1}, \\ &= (-1/2)e^{-2} - (-1/2)e^{2}. \end{aligned}$$

6. Compute:

$$\int_{0}^{1} \left(e^{3x} - \frac{1}{(x+1)^2}\right)\,dx$$

Solution.

$$\begin{aligned} \int_{0}^{1} \left(e^{3x} - \frac{1}{(x+1)^2}\right)\,dx &= \left[(1/3)e^{3x} + \frac{1}{x+1}\right]_{0}^{1}, \\ &= (1/3)e^{3} + \frac{1}{1+1} - (1/3)e^{0} - \frac{1}{1}. \end{aligned}$$

While it is fine to stop here, I will continue on,

$$\begin{aligned} (1/3)e^{3} + \frac{1}{1+1} - (1/3)e^{0} - \frac{1}{1} &= (1/3)e^{3} + \frac{1}{2} - (1/3) - 1, \\ &= (1/3)e^{3} - (5/6). \end{aligned}$$

7. Compute:

$$\int_{-2}^{-1} \left(x^2 - 2x^{-3} + 3\right)\,dx$$

Solution.

$$\begin{aligned} \int_{-2}^{-1} \left(x^2 - 2x^{-3} + 3\right)\,dx &= \left[\frac{x^3}{3} + x^{-2} + 3x\right]_{-2}^{-1}, \\ &= \frac{(-1)^3}{3} + (-1)^{-2} + 3(-1) - \left(\frac{(-2)^3}{3} + (-2)^{-2} + 3(-2)\right). \end{aligned}$$

8. What is the area under the curve $2/x$ between $x = 1$ and $x = 3$?

Solution. Now when the book says, "area under the curve," it really means, under the curve but above the x-axis. So now look at

$$\begin{aligned}\int_1^3 2/x\,dx &= \Big[2\ln(x)\Big]_1^3,\\ &= 2\ln(3) - 2\ln(1),\\ &= 2\ln(3),\\ &= \ln(9).\end{aligned}$$

9. What is the area under the curve $1/\sqrt{x}$ between $x = 1$ and $x = 2$?

Solution. Again, when the book says, "area under the curve," it really means, under the curve and above the x-axis. So now look at

$$\begin{aligned}\int_1^2 1/\sqrt{x}\,dx &= \Big[2x^{1/2}\Big]_1^2,\\ &= 2\sqrt{2} - 2.\end{aligned}$$

10. Suppose that during a controlled experiment, the temperature in a test tube at time t is rising at a rate of $6t^2 + 2$ degrees centigrade per minute. If the initial temperature is $0°$ C, what is the temperature in the test tube after 10 minutes?

Solution. Well this is just asking us to compute

$$\int \left(6t^2 + 2\right)\,dt = 2t^3 + 2t + K$$

for some constant K. Since the initial temperature is $0°$ C, $K = 0$. Hence after 10 minutes, the temperature is $2 \cdot 10^3 + 2 \cdot 10 = 2020$ degrees centigrade.