MATH 234: HOMEWORK 8

SOLUTIONS

1. Compute:

$$\int_0^2 \left(x^3 + 3x^2 + x + 1\right)\, dx$$

Solution. Write

$$\begin{aligned}\int_0^2 \left(x^3 + 3x^2 + x + 1\right)\, dx &= \left[\frac{x^4}{4} + x^3 + \frac{x^2}{2} + x\right]_0^2, \\ &= \frac{2^4}{4} + 2^3 + \frac{2^2}{2} + 2, \\ &= 16.\end{aligned}$$

2. Compute:

$$\int_{-100000}^{100000} x^3\, dx$$

Solution. Write

$$\begin{aligned}\int_{-100000}^{100000} x^3\, dx &= \left[\frac{x^4}{4}\right]_{-100000}^{100000}, \\ &= \frac{100000^4}{4} - \frac{(-100000)^4}{4}, \\ &= \frac{100000^4}{4} - \frac{100000^4}{4}, \\ &= 0.\end{aligned}$$

3. Compute:

$$\int_1^4 3\sqrt{x}\, dx$$

Solution. Write

$$\begin{aligned}\int_1^4 3\sqrt{x}\, dx &= \left[\frac{6}{3}x^{3/2}\right]_1^4, \\ &= 2 \cdot 4^{3/2} - 2 \cdot 1^{3/2}, \\ &= 16 - 2, \\ &= 14.\end{aligned}$$

4. Compute:

$$\int_2^3 (4x+4)\ dx$$

Solution. Write

$$\begin{aligned}\int_2^3 (4x+4)\ dx &= \left[\frac{4x^2}{2}+4x\right]_2^3, \\ &= 2\cdot 3^2+4\cdot 3-\left(2\cdot 2^2+4\cdot 2\right), \\ &= 18+12-8-8, \\ &= 14.\end{aligned}$$

5. Compute:

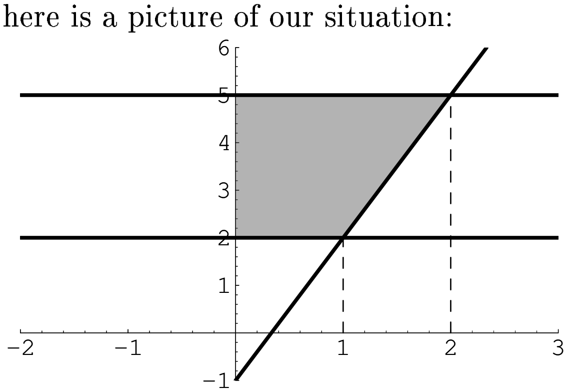
$$\int_1^e \frac{1}{x}\, dx$$

Solution. Write

$$\begin{aligned}\int_1^e \frac{1}{x}\, dx &= \Big[\ln|x|\Big]_1^e, \\ &= \ln(e)-\ln(1), \\ &= 1-0, \\ &= 1.\end{aligned}$$

6. Find the area of the region between $3x-1$, the y-axis and the lines $y=2$ and $y=5$.

Solution. So here is a picture of our situation:



Looking at the picture and doing a little algebra to completely convince us, we can see that we need to find

$$\begin{aligned}\int_0^1 (5-2)\ dx + \int_1^2 (5-(3x-1))\ dx &= \int_0^1 3\,dx + \int_1^2 (6-3x)\ dx,\\ &= 3 + \Big[6x - (3/2)x^2\Big]_1^2,\\ &= 3 + 6\cdot 2 - (3/2)2^2 - 6 + (3/2),\\ &= 3 + 12 - 6 - 6 + (3/2),\\ &= 3 + (3/2),\\ &= 9/2.\end{aligned}$$

7. Suppose that the profit realized by a department store t days after its opening is given by the formula $4t^3 - 2t + 1$. What was the average profit per day of the store during the first five days?

Solution. Well average profit is given by

$$\begin{aligned}\frac{1}{5-0}\int_0^5 \left(4t^3 - 2t + 1\right)\ dt &= \frac{1}{5}\Big[t^4 - t^2 + t\Big]_0^5,\\ &= \frac{1}{5}(5^4 - 5^2 + 5),\\ &= 5^3 - 5 + 1,\\ &= 121.\end{aligned}$$

Hence the average profit is \$121.

8. Suppose that a colony of fruit flies is growing exponentially with an annual growth constant 0.04. If there are currently 30000 flies present, what will be the average population over the next 6 months?

Solution. From previous chapters we know that the the number A of fruit flies is given by

$$A = 30000e^{0.04t}$$

where t is time in years. Hence the average population is

$$\begin{aligned}
\frac{1}{1/2-0}\int_0^{1/2} 30000e^{0.04t}\,dt &= 2\left[\frac{30000}{0.04}\cdot e^{0.04t}\right]_0^{1/2},\\
&= 2\left(\frac{30000}{0.04}\cdot e^{0.04\cdot(1/2)} - \frac{30000}{0.04}\cdot e^{0.04\cdot 0}\right),\\
&= \frac{2\cdot 30000}{0.04}(e^{0.04\cdot(1/2)}-1),\\
&= 1500000(e^{0.02}-1).
\end{aligned}$$

Hence, after 6 months we have about $1500000(e^{0.02}-1)$ fruit flies.

9. What is the consumers' surplus for the demand curve $p(x) = 5 - \frac{x}{20}$ at sales level $x = 60$?

Solution. Our formula for consumers' surplus says:

$$\begin{aligned}
\int_0^{60} (p(x)-p(60))\,dx &= \int_0^{60}\left(5-\frac{x}{20}-5+\frac{60}{20}\right)dx,\\
&= \int_0^{60}\left(-\frac{x}{20}+3\right)dx,\\
&= \left[-\frac{x^2}{40}+3x\right]_0^{60},\\
&= -\frac{60^2}{40}+3\cdot 60,\\
&= -90+180,\\
&= 90.
\end{aligned}$$

So we are left with a consumers' surplus of \$90.

10. Suppose that money is deposited steadily into a savings account at the rate of \$3000 per year. How long will it take for the balance to reach \$60000 if the account pays 4% interest compounded continuously?

Solution. By our formula for a future value of a continuous income stream, we see that

$$\begin{aligned}
60000 &= \int_0^N 3000e^{0.04(N-t)}\,dt, \\
&= \left[\frac{-3000}{0.04}e^{0.04(N-t)}\right]_0^N, \\
&= -75000 \cdot e^{0.04(N-N)} + 75000 \cdot e^{0.04(N-0)}, \\
&= 75000(e^{0.04N} - 1).
\end{aligned}$$

But this is true only when

$$\frac{60000}{75000} = e^{0.04N} - 1$$

which is true only when

$$1 + 4/5 = e^{0.04N}.$$

So we have

$$9/5 = e^{0.04N}.$$

Taking the natural logarithm of both sides we get

$$\ln(9/5) = .04N.$$

Hence, $N = 25\ln(9/5)$ years.