MATH 234: HOMEWORK 9

SOLUTIONS

1. Compute:

$$\int_0^3 \frac{x^2}{(2+3x^3)^2}\,dx$$

Solution 1. Write

$$\begin{aligned}
\int_0^3 \frac{x^2}{(2+3x^3)^2}\,dx &= \left[\frac{-1}{9(2+3x^3)}\right]_0^3, \\
&= \frac{-1}{9(2+3\cdot 3^3)} - \frac{-1}{9(2+3\cdot 0^3)}, \\
&= \frac{-1}{747} + \frac{1}{18}, \\
&= \frac{9}{166}.
\end{aligned}$$

Using u-substitution, set $u = 2+3x^3$.

Using guess and check, start with an initial guess of $(2+3x^3)^{-1}$ for the antiderivative.

2. Compute:

$$\int_0^{152} \frac{1}{\sqrt[3]{t+27}}\,dt$$

Solution 2. Write

$$\begin{aligned}
\int_0^{152} \frac{1}{\sqrt[3]{t+27}}\,dt &= \left[\frac{3}{2}(t+27)^{2/3}\right]_0^{152}, \\
&= \frac{3}{2}(152+27)^{2/3} - \frac{3}{2}(0+27)^{2/3}, \\
&= \frac{3}{2}(179)^{2/3} - \frac{3}{2}(27)^{2/3}, \\
&= \frac{3}{2}(179)^{2/3} - \frac{27}{2}.
\end{aligned}$$

Using u-substitution, set $u = t+27$.

Using guess and check, start with an initial guess of $(t+27)^{2/3}$ for the antiderivative.

3. Compute:

$$\int_0^2 \frac{30x}{(5x^2+3)^2}\,dx$$

Solution 3. Write

$$\begin{aligned}
\int_0^2 \frac{30x}{(5x^2+3)^2}\,dx &= \left[\frac{-3}{(5x^2+3)}\right]_0^2, \\
&= \frac{-3}{(5\cdot 2^2+3)} - \frac{-3}{(5\cdot 0^2+3)}, \\
&= \frac{-3}{23} - \frac{-3}{3}, \\
&= \frac{-3}{23} + 1, \\
&= \frac{20}{23}.
\end{aligned}$$

Using u-substitution, set $u = 5x^2 + 3$.

Using guess and check, start with an initial guess of $(5x^2+3)^{-1}$ for the antiderivative.

4. Compute:

$$\int_0^1 5x^4\sqrt{x^5+9}\,dx$$

Solution 4. Write

$$\begin{aligned}
\int_0^1 5x^4\sqrt{x^5+9}\,dx &= \left[(2/3)(x^5+9)^{3/2}\right]_0^1, \\
&= (2/3)(1^5+9)^{3/2} - (2/3)(0^5+9)^{3/2}, \\
&= (2/3)(10)^{3/2} - (2/3)(9)^{3/2}, \\
&= \frac{20\sqrt{10}-54}{3}.
\end{aligned}$$

Using u-substitution, set $u = x^5 + 9$.

Using guess and check, start with an initial guess of $(x^5+9)^{3/2}$ for the antiderivative.

5. Compute:

$$\int_0^1 xe^{(x^2)}\,dx$$

Solution 5. Write

$$\begin{aligned}\int_0^1 xe^{(x^2)}\,dx &= \left[\frac{e^{x^2}}{2}\right]_0^1, \\ &= \frac{e^{1^2}}{2} - \frac{e^{0^2}}{2}, \\ &= \frac{e}{2} - \frac{1}{2}.\end{aligned}$$

Note: My initial guess for the anti-derivative was $e^{(x^2)}$.

Using u-substitution, set $u = x^2$.

Using guess and check, start with an initial guess of $e^{(x^2)}$ for the antiderivative.

6. Compute:

$$\int_{-2}^{1} \frac{x}{\sqrt{x+3}}\,dx$$

Solution 6. Use integration by parts. Since

$$\int_a^b g(x)h'(x)\,dx = \Big[g(x)h(x)\Big]_a^b - \int_a^b g'(x)h(x)\,dx,$$

Letting $a = -2$ and $b = 1$, we'll set:

$$\begin{aligned} g(x) &= x & g'(x) &= 1 \\ h'(x) &= (x+3)^{-1/2} & h(x) &= 2(x+3)^{1/2} \end{aligned}$$

So now

$$\int_{-2}^{1} x(x+3)^{-1/2}\,dx = \Big[x \cdot 2(x+3)^{1/2}\Big]_{-2}^{1} - \int_{-2}^{1} 1 \cdot 2(x+3)^{1/2}\,dx.$$

Let's evaluate the right most integral:

$$\begin{aligned}\int_{-2}^{1} 2(x+3)^{1/2}\,dx &= \left[\frac{4}{3}(x+3)^{3/2}\right]_{-2}^{1}, \\ &= \frac{4}{3}(1+3)^{3/2} - \frac{4}{3}(-2+3)^{3/2}, \\ &= \frac{4}{3}(4)^{3/2} - \frac{4}{3}(1)^{3/2}, \\ &= \frac{32}{3} - \frac{4}{3}.\end{aligned}$$

And

$$\left[x \cdot 2(x+3)^{1/2}\right]_{-2}^{1} = 1 \cdot 2(1+3)^{1/2} - (-2) \cdot 2(-2+3)^{1/2},$$
$$= 8.$$

Putting it all together we get

$$\int_{-2}^{1} \frac{x}{\sqrt{x+3}}\, dx = 8 - \left(\frac{32}{3} - \frac{4}{3}\right) = \frac{-4}{3}.$$

7. Considering

$$\lim_{b \to \infty} \frac{2b-1}{b},$$

which of the following is true?

(A) The limit exists and is equal to zero.
(B) The limit exists and is equal to one.
(C) The limit exists and is equal to two.
(D) The limit diverges.
(E) None of the above.

Solution 7. Well

$$\lim_{b \to \infty} \frac{2b-1}{b} = \lim_{b \to \infty} \frac{2 - 1/b}{1} = 2.$$

Hence (C) is right.

8. Compute:

$$\int_{3}^{\infty} e^{-x/2}\, dx$$

Solution 8. Write

$$\int_{3}^{\infty} e^{-x/2}\, dx = \lim_{b \to \infty} \int_{3}^{b} e^{-x/2}\, dx,$$
$$= \lim_{b \to \infty} \left[-2e^{-x/2}\right]_{3}^{b},$$
$$= \lim_{b \to \infty} \left(-2e^{-b/2} + 2e^{-3/2}\right),$$
$$= 2e^{-3/2}.$$

9. Find the value of k that makes $f(x) = kx$ a probability function on the interval $1 \leq x \leq 2$.

Solution 9. We need to find k such that kx is non-negative for $1 \leq x \leq 2$, and such that

$$\int_1^2 kx\,dx = 1.$$

Well,

$$\int_1^2 kx\,dx = \left[\frac{k}{2}x^2\right]_1^2 = \frac{k}{2}2^2 - \frac{k}{2}1^2 = k(2 - \frac{1}{2}) = k \cdot \frac{3}{2}.$$

So if $k = 2/3$, then

$$\int_1^2 kx\,dx = 1.$$

Note that $k = 2/3$ also makes $f(x)$ non-negative for $1 \leq x \leq 2$.

10. A random variable X has a cumulative distribution function

$$F(x) = 1 - \frac{1}{x^2}$$

for $x \geq 1$. Find $\Pr(a \leq X \leq 5)$.

Solution 10. Well this is just

$$F(5) - F(a) = 1 - \frac{1}{5^2} - (1 - \frac{1}{a^2}) = \frac{1}{a^2} - \frac{1}{25}.$$