Final Exam Review Sheet

December 10, 2004

This is <u>not</u> intended to be a <u>complete</u> study guide, only to point out key ideas and facts.

1. (1.2 -1.8, 3.1, 3.2, 4.3, 4.5) Be able to differentiate important functions:

   (a) Remember the original limit definition of the derivative, and know what the derivative of a function is.

   (b)

$$\frac{d}{dx}(x^r) = rx^{r-1}$$

Power Rule

   (c)

$$\frac{d}{dx}\Big(f(x)^r\Big) = rf(x)^{r-1}f'(x)$$

General Power Rule

(d)

$$\frac{d}{dx}\Big(f(x)g(x)\Big) = f(x)g'(x) + f'(x)g(x)$$

Product Rule

(e)

$$\frac{d}{dx}\Big(\frac{f(x)}{g(x)}\Big) = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$$

Quotient Rule

(f)

$$\frac{d}{dx}(e^{f(x)}) = e^{f(x)}f'(x)$$

Instances of the Chain Rule

(g)

$$\frac{d}{dx}(\ln f(x)) = \frac{1}{f(x)}f'(x).$$

2. Know what it means for a function to be continuous (1.5).

3. First and Second Derivative Tests for one variable, and applications (2.1, 2.2, 2.3, 2.5, 2.6).

FDT:

(a) Possible relative maxima and minima satisfy $f'(x) = 0$.

(b) If $f'(x) = 0$, we can verify that x corresponds to a relative minimum, maximum or neither by observing if (and how) $f'(x)$ changes sign at x.

SDT: If $f'(x) = 0$,

(a) $f''(x) > 0 \;\Rightarrow$ concave up $\Rightarrow$ rel. min.

(b) $f''(x) < 0 \;\Rightarrow$ concave down $\Rightarrow$ rel. max.

(c) $f''(x) = 0 \;\Rightarrow$ test fails; possible inflection point.

4. **Optimization in Economics:** (2.7)

- Demand function: $p = D(x)$ ($x =$ quantity, $p =$ price
- Cost function: $C(x)$
- Revenue function $R(x) = x \cdot p = xD(x)$
- Profit function: $P(x) = R(x) - C(x)$

5. **Implicit Differentiation and Related Rates:** (3.3)

- Draw a picture if possible
- Assign variables to quantities that vary with respect to time
- Find an equation relating these variables
- Differentiate implicitly with respect to t

- Substitute what it know and solve for what is requested

6. **Exponential Growth and Decay and Compound Interest:** (4.1, 5.1, 5.2)

$$P(t) = P_0 e^{kt}.$$

$k =$ growth rate and $P_0 =$ initial amount.

Be able to answer questions related to how to find these constants or answer related questions.

7. Know what the relative (or percent) rate of change of a function is (and how to find it) (5.3).

8. **Elasticity of Demand:** (5.3)
$p =$ price, $q =$ quantity, $q = f(p)$,

$$E(p) = \frac{-pf'(p)}{f(p)}.$$

- Demand is elastic at p_0 if $E(p_0) > 1 \Rightarrow (R \uparrow \Leftrightarrow p \downarrow)$

- Demand is inelastic at p_0 if $E(p_0) < 1 \Rightarrow (R \uparrow \Leftrightarrow p \uparrow)$

9. Know what the integral represents in terms of antiderivatives and in terms of areas. (6.1 - 6.3)

10. Be able to integrate key functions and apply techniques:

    (a)

$$\int x^r \; dx = \frac{1}{r+1} x^{r+1} + C$$

(b)

$$\int \frac{1}{x}\ dx = \ln|x| + C$$

(c)

$$\int e^{kx}\ dx = \frac{1}{k}e^{kx} + C$$

(d) u substitution (9.1)

(e) Integration by parts (9.2)

$$\int u\ dv = uv - \int v\ du$$

(f) Improper integrals (9.6)

11. Using probability density functions (know what these are and their characteristics) to find probabilities. (12.1 - 12.2)

12. **Applications of the Definite Integral:** (6.5)

- Average value of a function over $[a, b]$:

$$\frac{1}{b-a}\int_a^b f(x)\ dx$$

- Consumer Surplus:

$$\int_0^q D(x)\ dx - qp$$

- Producer Surplus:

$$qp - \int_0^q S(x)dx$$

13. Know how to find partial derivatives, and what they represent. (7.1 - 7.2)

14. **First and Second Derivative Tests for Two Variables:** (7.3)

FDT: Possible relative maxima and minima satisfy

$$\frac{\partial f}{\partial x}(a,b) = 0 \qquad \frac{\partial f}{\partial y}(a,b) = 0$$

SDT: Let

$$D(x,y) = \left(\frac{\partial^2 f}{\partial x^2}\right)\left(\frac{\partial^2 f}{\partial y^2}\right) - \left(\frac{\partial^2 f}{\partial x \partial y}\right)^2$$

(a) $D(a,b) > 0$, $\frac{\partial^2 f}{\partial x^2}(a,b) > 0 \Rightarrow$ relative min.

(b) $D(a,b) > 0$, $\frac{\partial^2 f}{\partial x^2}(a,b) < 0 \Rightarrow$ relative max.

(c) $D(a,b) < 0 \Rightarrow$ neither

(d) $D(a,b) = 0 \Rightarrow$ test fails

15. **Lagrange Multipliers:** (7.4)

Goal: Maximize or minimize $f(x, y)$ subject to the constraint $g(x, y) = 0$.

(a) Define

$$F(x, y, \lambda) = f(x, y) + \lambda g(x, y)$$

(b) solve

$$\begin{cases} \frac{\partial F}{\partial x} = 0 \\ \\ \frac{\partial F}{\partial y} = 0 \\ \\ \frac{\partial F}{\partial \lambda} = 0 \end{cases}$$