Math 234
Review - Test 3

The usual disclaimer: This is *NOT* intended to be a complete study guide. You should consult Tim's lecture notes for a complete list of topics to be covered on the exam. This is just a list of some of the common mistakes made on the types of problems you might find on exam 3.

Antiderivatives/Integrals - Overview

Most of what you've been doing for the last several weeks boils down to one thing: Integrals. Since you won't know which section an integral is from on the exam, here is the general approach I would take:

- Check first to see if it's an "easy" integral, one you can do without having to work to hard. Basically, those are integrals that look like
$$\int x^r\,dx,\ \int e^x\,dx,\ \int \frac{1}{x}dx$$
(perhaps with small variations) or sums of functions of these types. For example, even though it looks awful,
$$\int (4e^{\pi x} - \frac{367}{x} + \frac{9}{5}x^{\frac{6}{7}} + 7.935x^{-\frac{3}{4}})dx$$
is an "easy" integral, because it's a sum of the three types of functions I listed above.
- Clues that it is not an "easy" integral:
  - if it is the product of two functions of the types above, like xe^x or $2x^2(x^3+4)$
  - if it is the quotient of two functions of the types above, like $\frac{x}{(3x^2-1)^{1/2}}$
  - if it has a "$\ln x$" anywhere in it - you don't know how to integrate $\ln x$.
- If it isn't an easy integral, you're going to need u-substitution or integration by parts. There (unfortunately) is no good way to tell which one, unless Tim is kind enough to tell you which one to use, as on the quiz. Assuming he doesn't, I would try u-substitution first, since it's a little bit easier. If things don't fall into place, give integration by parts a try.

Regardless of the method used to integrate, remember:

- If it's an indefinite integral, DON'T FORGET YOUR "$+C$"!!! I would write a big fat "$+C$" on the front of your test when you get it so you'll remember before you hand it in to go back and make sure you didn't leave any off. I would be ecstatic if not a single person lost a point for forgetting one.
- If it's a definite integral, remember that the top number get filled in first, then the bottom. Be careful about distributing your negative signs!
- Remember $\int \frac{1}{x}dx = \ln|x| + C$ - the absolute values are NECESSARY!!

Integration by u-substitution

Use the step-by-step procedure I gave you in class to work through these.

Remember that if there are constants running around that don't match up, that doesn't matter! Those are fixable.

Don't forget to put every back in terms of x's when you're done.

Integration by parts

Again, I recommend the procedure I gave you in class - it's *similar* to the one for u-substitution, but make sure you know what the differences are.

Know the formula!

A common mistake: You DO have to take an integral at some point! After you've made all your substitutions and plugged everything into the formula, I know it seems like you've worked so hard that you should be done, but you're NOT! You still have an integral there, and you do have to actually integrate at this point.

Improper Integrals

I recommend the two-step process: (1) Find $\int_0^b f(x)dx$ (if, say, 0 is the lower limit on your integral), and (2) Find $\lim_{b\to\infty} \int_0^b f(x)dx$. This way, you only have to worry about carrying your limit through on part (2) - fewer chances to make a mistake.

DO NOT write anywhere that $\int_0^\infty f(x)dx = \int_0^b f(x)dx$ - this is just plain false!

When you're actually "substituting ∞" into the result of your integration, remember there are pretty much two choices on a term that has a b in it: either the b is in the denominator, and that term is going to go to 0 (which means only the non-b terms contribute to the value of the limit), or there's a b in the numerator, and that term is increasing without bound (which means the integral diverges).

Remember that a limit can't EQUAL ∞ - instead, say it diverges.

Applications of Integrals (section 6.5)

Make sure you know the formulas you need:

- average value
- consumers' surplus
- producers' surplus

These types of problems are straightforward if you know and understand these formulas.

Don't get consumers' and producers' surplus mixed up! If you get a negative answer, you probably have the wrong formula.

Probability

Pretty much everything you need to know can be summed up in example 6 on page 621.

You need to check TWO things to verify that a function is a probability density function. The hard part is checking that the integral is 1, but you also MUST check that the function is always nonnegative.

To find the cumulative distribution function, you need to integrate the probability density function, THEN find the correct value of "C". You want the C that makes $F(A) = 0$, where A is the lower limit on your variable X.

Remember: if you're finding a probability (i.e. $\Pr(1 \leq X \leq 2)$ or something similar), and you get an answer that is either negative or greater than 1, PLEASE PLEASE PLEASE go back and check your work, because PROBABILITIES ARE ALWAYS BETWEEN 0 AND 1. I promise.

Functions of Several Variables

We'll go over these in class Wednesday and Monday.

Study Hints

Even more than the previous exams, your success on this exam is going to be determined by how many practice problems you do. If it takes you five minutes to find $\int x^{-3/2}dx$, you're never going to have time to finish the whole exam. Practice, practice, practice. Integration, u-substitution, integration by parts, partial derivatives (section 7.2) - all of these are just computation, and the ONLY way to get good at them is to do a zillion of them.

Check out questions 19 through 36 on page 498. It's a mix of u-substitution and integration by parts questions - it's a good way to practice integrating when you don't know which technique to use.