1.2: Lines in the Plane

Suggested exercises: 1-29 odd, 41-45 odd
Lines are among the simplest things we can draw. But mathematically, they allow us to represent a lot of situations and solve interesting problems. For this reason, we will consider a few different ways of writing lines, and how to go between them.

4 keys:

- Linear equations
- Slope of a line
- Equations of lines
- Applications

I Linear equations

Definition A **linear equation** is an equation of the form

$$Ax + By = C,$$

where x and y are variables and A, B, and C are constants. Both A and B cannot both be zero.

Intercepts

x-intercept The point of the form $(a, 0)$ where the line crosses the x-axis

y-intercept The point of the form $(0, b)$ where the line crosses the y-axis

Example.

Example.

II Slope of a line

Definition The **slope** m of a line is an indication of how fast it is moving up or down. If (x_1, y_1) and (x_2, y_2) are two points on a line, the slope is defined as

$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$

We can think of this as

$$slope = \frac{rise}{run}.$$

Example.

Example.

III Equations of lines

Definition The **point-slope form** of a line with slope m passing through the point (x_1, y_1) is

$$y - y_1 = m(x - x_1).$$

Example.

Definition The **slope-intercept form** of a line with slope m and x-intercept $(0, b)$ is

$$y = mx + b.$$

Example.

Property 1 Suppose that the line L_1 has slope m_1 and the line L_2 has slope m_2. Then

1. L_1 and L_2 are parallel if and only if $m_1 = m_2$.
2. L_1 and L_2 are perpendicular if and only if $m_1 = \frac{-1}{m_2}$ (or equivalently, $m_2 = \frac{-1}{m_1}$).

Example.

IV Applications

Example.

Example.