1.3 Systems of Equations in Two Variables

Suggested exercises: 1-29 odd
Lines are used to describe different occurrences in manufacturing, business, and industry. Sometimes the places where two or more lines intersect have an important meaning. The set of these lines forms a *linear system*, and we will examine ways to find where these lines intersect.

3 keys:

- Solutions of linear systems
- Method of elimination
- Applications

I Solutions of linear systems

Definition A **system of linear equations** in two variables x and y is a set of lines

$$\begin{aligned} A_1x + B_1y &= C_1 \\ A_2x + B_2y &= C_2 \\ &\vdots \end{aligned}$$

Definition A **solution** of a linear system is a pair (x_0, y_0) that satisfies all of the equations in the linear system.

3 possibilities:

1. No solutions

   Example.

2. Infinitely many solutions

   Example.

3. Exactly one unique solution

   Example.

Definition A linear system with no solutions is called **inconsistent**, and a system with one or more solutions is called **consistent**.

II Method of elimination

We can get an idea of the nature of the solutions to our linear system by graphing the lines, but using algebra we can find out precisely what the solutions are (if they exist). The most common algebraic method is called the *method of elimination.*

Definition The **method of elimination** algebraically determines whether a linear system is consistent, and gives the solution(s) if it is consistent. 3 steps:

1. *Eliminate one of the variables.* Choose a variable and multiply the equations by something so that that variable has the same coefficient in each equation. Then subtract one equation from the others, giving you only one variable.
2. *Solve this new equation with only one variable.*
3. *Solve for the other variable.* Plug the value for the variable from the last step into one of the original lines, and then solve for the remaining variable. If this equation can't be solved, the system is inconsistent. Otherwise, you get the solution.

Example.

Example.

Example.

III Applications

There are two applications we will consider that use linear systems. They concern production and profit. In each case, the solution of the linear system gives us an answer to a question with several different conditions.

Mixing problems

In **mixing problems**, we have two different types of material that go into our products, in different ratios. We would like to use all of our raw materials to make the different products, with nothing left over.

1. *Find the products and the materials in the problem.* Use a variable for each product, representing how much of that product will be made. The coefficients will be the amount of the material used in that product.
2. *Find the ratios of the materials in each product.* We will form a table, with each material in a row and each product in a column. The ratios of the materials form the coefficients of each variable in the columns.
3. *Write down the linear system from this table.*
4. *Solve the linear system.*

Example. Oats and wheat are to be blended into 2 different types of animal feed. Feed A has 60 lb. of oats and 15 lb. of wheat per 100 lb. of feed, while feed B uses 35 lb. of oats and 40 lb. of wheat per 100 lb. of feed. Suppose there are 14,550 lb. of oats and 8250 lb. of wheat available. Find a linear model that describes the use of oats and wheat in the two feeds if all the materials are to be used. Find a combination of the two feeds that will satisfy this model.

Example. An electrical company uses two different mixtures of gold dust and silver dust in their products. The premium mixture uses 7 oz. of gold dust for every 3 oz. of silver, and the standard mixture uses 4 oz. of gold dust for every 6 oz. of silver. The company wishes to prepare mixtures that will exhaust its current stock of 250 oz. of gold and 300 oz. of silver. How much of each type of mixture should be made?

Breakeven analysis

In **breakeven analysis**, we wish to examine how much we need to sell before we start making a profit. A business will typically have costs that are *fixed*, meaning they don't depend on how many products we make. There also *unit costs* associated with production of each product. Our $t$otal cost is a combination of these two types of cost. Finally, we have a unit price, the price that we use to sell the product, which determines our $r$evenue. Our goal is to find out how many products we need to sell so that our total cost equals our revenue.

We introduce variables to represent each of these quantities:

- x - the number of units we sell
- F - fixed costs
- V - unit cost
- P - price per unit
- y - the variable we use to represent both the total cost and the revenue

These variables are related in the following way, giving our linear system:

$$\begin{aligned} y &= F + Vx \\ y &= Px \end{aligned}$$

Solving this linear system tells us when cost is equal to revenue. The variable x will tell us how many units we need to sell, and y tells us how much revenue/cost we have at that point.

Example. A candy shop has a fixed cost of $550 per week and a per unit cost of $2 per pound of candy. If the candy sells for $3 per pound, find the breakeven point for the shop. What is the total revenue at the breakeven point?

Example. A shoe company has fixed costs of $5600 per week and a unit cost of $13 per pair of shoes. The company sells the shoes at a wholesale price of $20 per pair. Find the breakeven point and the revenue at that point.