1.5 Linear Programming in the Plane

Suggested exercises: 3, 5-7, 9-25 odd
A family of lines with the same slope is given by a *linear function*. In many of the applications of linear equations that we will examine, we would like to find the maximum or minimum value of a linear function within a certain set of constraints. The methods that are used in this type of problem are a part of what is called *linear programming*.

3 keys:

- Linear functions
- Maximum and minimum values
- Linear programming

I Linear functions

Given a line $Ax + By = C$, changing C doesn't change the slope of our line – only the x- and y-intercepts. Instead of writing C we will write z and think of this as a function that takes (x, y) and gives us z.

Definition An equation of the form $z = Ax+By$ is a **linear function**. The lines corresponding to particular values of z are called **lines of constancy** for the linear function.

Note that all the lines of constancy for a linear function are parallel since the slope doesn't change.
Example.

Example. A company markets 2 products. Product A sells for \$12 per unit, and product B sells for \$8 per unit. Suppose that x units of Product A and y units of product B are sold each day. Express the daily revenue R generated by the sales as a linear function of x and y and find the sales combinations that result in a revenue of $R = \$2000$.

II Maximum and minimum values

The lines of constancy are so named because any for any value of x and y on that line, the linear function takes the same value. If we have a region in the plane defined by some inequalities, we can try to maximize or minimize our linear function in that region.
Example.

Property 1 Let $z = ax + by$ be a linear function and P a polygon in the plane. Then the maximum value and the minimum value of z are attained at corner points of P.

Example.

III Linear programming

Definition Minimizing or maximizing a linear function when the variables are restricted by a system of linear equations is called a **linear program**. The linear equations, or **constraints**, can be either equalities or inequalities, and determine the **feasibility region** for the linear program. The function to be optimized is called the **objective function**.

Example.

Example.