1.6 Applications of Linear Programming

Suggested exercises: 1-15 odd
In this section we use the tools of linear programming discussed in the last section to solve some applications. We will consider three different diverse problem areas, but they can all be solved in a similar manner once we interpret them as linear programs.

4 keys:

- Basic setup
- Production schedules
- Maximizing revenue
- Diet problem

I Basic setup

There are some basic steps that can be used to solve several problems, including all of the ones in this section.

1. *Name and define the appropriate variables.*
2. *Determine the objective function and express it in terms of the variables.*
3. *Convert each condition that constrains the variables into an inequality.* (Look for "at least" and "at most".)
4. *Use linear programming to find the solution.* Remember to interpret the final result in terms of the original problem.

II Production schedules

A *production schedule* involves using a set of resources to produce products at minimum cost. The goal is to find out how long we must use our factories to meet a certain production goal.

The variables will typically represent the length of time that we use our facilities, the objective function expresses the cost of using these facilities, and the constraints will involve things like contracts for a certain amount of product to be made, sometimes within a limited time.

Example. The Carbon Coal Company has 2 mines, a surface mine and a deep mine. It costs $200 a day to operate the surface mine and $250 a day to operate the deep mine. Each mine produces a medium grade and a medium-hard grade of coal, but in different proportions. The surface mine produces 12 tons of medium grade and 6 tons of medium-hard grade coal a day, and the deep mine produces 4 tons of medium grade and 8 tons of medium-hard grade coal a day. The company has a contract to deliver at least 600 tons of medium grade and 480 tons of medium-hard grade coal within 60 days. How many days should each mine be operated so that the contract can be filled at minimum cost?

III Maximizing revenue

As in section 1.3, we can use linear programming to determine how best to use our limited resources to *maximize revenue*. We will start with a pair of products that we can make, and a limited amount of materials for each. The goal is to determine how many of each product to make to maximize revenue.

The variables will represent the quantity of the products to be made, the objective function gives the total revenue, and the constraints will be the limited amount of our materials. We also use our "common sense" constraints, like the fact that we can't produce a negative number of products.

Example. The Easy Mix Cement Company produces their Red Seal and Black Seal cements from a mixture of ground limestone, clay, and shale. The Red Seal mixture uses 1 part of limestone to 1 part clay and 2 parts shale, whereas the Black Seal mixture requires 4 parts limestone to 1 part clay and 1 part shale. There are 8 tons of limestone, 3.5 tons of clay, and 6 tons of shale on hand. Red Seal sells for $200 a ton, while Black Seal sells for $250 a ton. Assuming that the company can sell all that they produce, set up and solve a linear program to find the amount of each mixture that should be formed for maximum revenue.

IV Diet problem

For a change of pace, we can leave industry and consider putting together a diet. Given a set of foods and nutrients, how can we achieve our dietary goals while minimizing our cost?

The variables will be the amount of each food to include in the diet, the objective function is the cost of the foods, and the constraints will involve getting enough of each nutrient, as well as some practical limits on the amount of each food to eat.

Example. We are given 2 foods, bread and milk, and we want to consider 2 nutrients, protein and calories. Suppose that 1 slice of bread has 1 g. of protein and 60 calories, and that 1 cup of milk has 9 g. of protein and 90 calories. Further, suppose that the cost of bread is $.02 per slice and the cost of milk is $.10 per cup. We restrict the diet to at most 100 slices of bread and at most 40 cups of milk. Find a daily bread-milk diet that contains at least 90 g. of protein and 2700 calories such that the cost is minimized.