2.2 More on Gaussian Elimination (Part One)

Suggested exercises: 1-9 odd
When we have more than two variables and two equations, using the method of elimination gets too difficult and confusing. We use Gaussian elimination with a matrix to make our work easier, and then we use back substitution to arrive at our answer. If we add a few steps after Gaussian elimination, we can get our solution even more easily. This will be called *back addition.*

2 keys:

- Back addition
- Reduced echelon form

I Notation

When we describe our elementary row operations on a matrix, we can use a nice shorthand to save space. You should learn how to describe any elementary row operation using this notation.

Here are examples demonstrating the notation.

1. $\mathbf{R_1} \leftrightarrow \mathbf{R_2}$: Swap row 1 and row 2
2. $\mathbf{R_2'} = \mathbf{3R_2}$: The new row 2 is found by multiplying the old row 2 by 3.
3. $\mathbf{R_3'} = \mathbf{R_3} - \mathbf{5R_1}$: The new row 3 is the old row 3 minus 5 times row 1.

Note that we use a $'$ symbol to tell the old rows and the new row apart. See page 72 in the book for more details.

II Back addition

After we finish the steps of Gaussian elimination, we can turn our matrix back into equations and solve for our variables. We can also work with the matrix for another few steps to solve for our variables instead. This is called back addition.

To perform back addition:

1. Use Gaussian elimination to put the matrix in row echelon form.
2. Beginning with the last nonzero row, use the elementary row operations to turn all the entries above the leading 1 into zeros.
3. Move up to the next row with a leading 1 and repeat Step 2. Stop when all the leading 1s have zeros above and below them.

After doing this, we write the equations from our matrix and solve for the variables.

Example.

III Reduced echelon form

When we use Gaussian elimination, we call the form of our new matrix the *row echelon form.* Unfortunately, this form is not unique – someone else could do Gaussian elimination differently and get a different matrix.

After we perform back addition, the matrix is even nicer. Not only is it more useful for solving our equations, this form is also unique – anyone who starts with the same matrix and uses Gaussian elimination and back addition will get the same answer. We call this *reduced echelon form.*

Definition A matrix is said to be in **reduced echelon form** if, in addition to having the properties of row echelon form, it also has the property that "all entries above any leading 1 are zero".

Example.

Example.