6.2 Calculation of Probabilities

Suggested exercises: 1-3, 9-11, 19, 21, 23-25, 29-35 odd
We will use what we know about set theory to calculate probabilities. For instance, we will describe more complicated events in terms of simple events, and then apply some basic properties of probability to compute the probability of all of the events.

5 keys:

- Additivity property
- Complementary events
- Relative complements
- Inclusion-exclusion
- Venn diagrams

I Additive property

Definition Two events E and F are *mutually exclusive* if they are disjoint subsets of the sample space, or $E \cap F = \emptyset$.

> **Property 1 (Additive property)** If E and F are mutually exclusive events, so $E \cap F = \emptyset$, then
>
> $$\Pr(E \cup F) = \Pr(E) + \Pr(F).$$

Example.

Definition The events $E_1, E_2, \ldots, E_n$ are *mutually exclusive* if $E_i \cap E_j = \emptyset$ whenever $i \neq j$.

> **Property 2 (Additive property)** If $E_1, E_2, \ldots, E_n$ are mutually exclusive events, then
>
> $$\Pr(E_1 \cup E_2 \cup \cdots \cup E_n) = \Pr(E_1) + \Pr(E_2) + \cdots + \Pr(E_n).$$

Example.

II Complementary events

Definition The *complementary event* E' of an event E in the sample space S is the event

$$E' = \{s \in S : s \notin E\}.$$

Note: we have $E \cap E' = \emptyset$ and $E \cup E' = S$.

Property 3 (Complementation property) For any event E, the complementary event E' has probability

$$\Pr(E') = 1 - \Pr(E).$$

Example.

Example.

III Relative complements

Definition If E and F are events in a sample space S, the event

$$E \cap F' = \{s \in E : s \notin F\}$$

is called the *relative complement* of F in E. It is the outcomes that are in E but not in F.

Property 4 (Relative complements) For any events E and F, we have

$$\Pr(E \cap F') = \Pr(E) - \Pr(E \cap F).$$

Example.

IV Inclusion-exclusion

When two events are not mutually exclusive, we cannot apply the Additive property. Instead, we need to take into account the outcomes in the intersection of the two events. We have already seen this principle: it is inclusion-exclusion.

Property 5 (Inclusion-exclusion property) For any events E and F, we have

$$\Pr(E \cup F) = \Pr(E) + \Pr(F) - \Pr(E \cap F).$$

Example.

Example.

V Venn diagrams

Instead of using inclusion-exclusion when there are more than two events, we will use Venn diagrams to calculate probabilities.

Example.

VI Further examples

Example.

Example.