7.2 Expected Value

Suggested exercises: 1-21 odd
A random variable can take several different values. But the probability of each value is not the same; that is, we don't expect to get each value equally likely. If we repeat an experiment several times, what value can we expect to get on average? This value is called the *expected value.*

4 keys:

- Expected value of a random variable
- Mean of a binomial distribution
- Sampling without replacement
- Sample mean

I Expected value of a random variable

Definition Let X be a random variable whose values are $x_1, x_2, \ldots, x_n$ and $p_j = Pr(X = x_j)$ the probability that X takes the value x_j. Then the **expected value** or **expectation** of the random variable X is denoted $E(X)$ and defined to be

$$E(X) = x_1p_1 + x_2p_2 + \cdots + x_np_n.$$

This is also called the **mean** of X and denoted by μ (the Greek letter *mu*), so $E(X) = \mu$.

Example.

Example.

II Mean of a binomial distribution

For binomial distributions, our simple formula for probability distributions gives us a simple formula for the mean.

> **Property 1** (Mean of a binomial distribution) Suppose a Bernoulli process of n trials has a probability of success p on one trial. Then the number of successes has a binomial distribution with mean
>
> $$\mu = np.$$

Example.

Example.

III Sampling without replacement

Sampling without replacement means our sample space changes with each selection. This will change our probability. In a special case, we have a nice formula.

> **Property 2** Suppose an urn has a red balls and b blue balls, for a total of $a+b$ balls. If a sample of n balls is drawn without replacement, then the expected number of red balls is
>
> $$n \cdot \frac{a}{a+b}.$$

Example.

IV Sample mean

Even if we don't know the theoretical probabilities of an event, we can still try to find the mean based on a given sample. Here we assume that our outcomes occur with roughly their actual probabilities. This makes it dangerous to make too broad of a conclusion based on a small sample.

Definition If the values $x_1, x_2, \ldots, x_n$ are obtained from a sample (population), then the **sample mean** is denoted by $\overline{x}$ and defined by

$$\overline{x} = \frac{x_1 + x_2 + \cdots + x_n}{n}.$$

Example.

Further examples

Example.