Math 241 **Practice Exam 3** **Spring 2009**

Show all your work. No credit will be given for unjustified answers. Calculators are not allowed. Good luck!

1. (10 points) Sketch the region of integration and then reverse the order of integration:

$$\int_0^2 \int_0^{4x-x^2} f(x,y)\,dy\,dx.$$

2. (10 points) Find the mass of the lamina bounded by the circle $x^2+(y-1)^2=1$ with density $\delta(x,y)=(x^2+y^2)^{-1/2}$. (Use polar coordinates.)

3. (10 points) The solid T is bounded by the surfaces $z=0$, $y=x^2$, and $2z+y=4$. Sketch the region T.

   (a) (5 pts) Sketch the projection of T to the xy-plane and express $\int\int\int_T f(x,y,z)\,dV$ as an iterated integral in the order $dz\,dy\,dx$.

   (b) (5 pts) Sketch the projection of T to the yz-plane and express $\int\int\int_T f(x,y,z)\,dV$ as an iterated integral in the order $dx\,dy\,dz$.

4. (10 points) Let T be the solid in the first octant ($x \geq 0$, $y \geq 0$, $z \geq 0$) above the cone $z=\sqrt{3x^2+3y^2}$ and inside the sphere $x^2+y^2+z^2=1$.

   (a) (5 pts) Convert $\int\int\int_T \sqrt{x^2+y^2+z^2}\,dV$ to the cylindrical coordinates, but **do not evaluate**.

   (b) (5 pts) Convert $\int\int\int_T \sqrt{x^2+y^2+z^2}\,dV$ to the spherical coordinates and **evaluate**.

5. (10 points) Let R be the region bounded by the coordinate axes and the line $x+y=1$. Evaluate

$$\int\int_R e^{(x+y)^2}\,dA$$

   by the substitution $u=x+y$, $v=x-y$.

6. (10 points)

   (a) Determine whether the vector field $\mathbf{F}(x,y)=\langle x-\frac{y}{x^2}, \frac{1}{x}+3y^2+1\rangle$ is conservative in the region $D=\{(x,y): x>0\}$ and if so, then find a potential function.

   (b) Evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$, where C runs from $(1,1)$ to $(1,-1)$ along the arc of the circle $x^2+y^2=2$ in the right half-plane.