Math 221 Quiz #2 Solutions

1. (a) Complete the following sentence with a careful and complete definition:

A sequence (a_n) of real numbers *converges* to a real number L if

for all $\epsilon > 0$ there exists a natural number N so that if $n \geq N$ then $|a_n - L| \leq \epsilon$.

(b) Prove carefully **from the definition** that $\lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$.

ANSWER: Let $\epsilon > 0$. Choose N so large that $N > \frac{1}{\epsilon^2}$. Then $\frac{1}{N} < \epsilon^2$ and $\frac{1}{\sqrt{N}} < \epsilon$. If $n \geq N$, then

$$|\frac{1}{\sqrt{n}} - 0| = \frac{1}{\sqrt{n}} \leq \frac{1}{\sqrt{N}} < \epsilon.$$

Thus $\lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$. This finishes the proof.