

Math 221 Quiz #3 Solutions

1. Match each series in the left hand column with a name in the right hand column. For each series, circle either *convergent* or *divergent* as appropriate.

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|--|------------------------|
| (i) $\sum_{i=0}^{\infty} 4\left(-\frac{1}{3}\right)^i$ | (a) harmonic series |
| (ii) $\sum_{i=1}^{\infty} \frac{1}{i}$ | (b) geometric series |
| (iii) $\sum_{i=0}^{\infty} (-1)^i \frac{1}{i^2+1}$ | (c) alternating series |

Answers: (i) = (b) and/or (c) (both answers were accepted) convergent divergent

(ii) = (a) convergent divergent

(iii) = (c) convergent divergent

2. Is the series

$$\sum_{i=0}^{\infty} \left(\frac{1}{3^i} - \frac{1}{5^i} \right)$$

convergent or divergent? If convergent, give the value of the sum.

ANSWER: The series $\sum_{i=0}^{\infty} \frac{1}{3^i} = \sum_{i=0}^{\infty} \left(\frac{1}{3}\right)^i$ and $\sum_{i=0}^{\infty} \frac{1}{5^i} = \sum_{i=0}^{\infty} \left(\frac{1}{5}\right)^i$ are both convergent geometric series. From the general formula $\sum_{i=0}^{\infty} ar^i = \frac{a}{1-r}$, valid for $r < 1$, we find

$$\sum_{i=0}^{\infty} \frac{1}{3^i} = \frac{1}{1 - \frac{1}{3}} = \frac{3}{2}$$

and

$$\sum_{i=0}^{\infty} \frac{1}{5^i} = \frac{1}{1 - \frac{1}{5}} = \frac{5}{4}.$$

Thus

$$\sum_{i=0}^{\infty} \left(\frac{1}{3^i} - \frac{1}{5^i} \right) = \sum_{i=0}^{\infty} \frac{1}{3^i} - \sum_{i=0}^{\infty} \frac{1}{5^i}$$

is also convergent, and its sum is $\frac{3}{2} - \frac{5}{4} = \frac{1}{4}$.