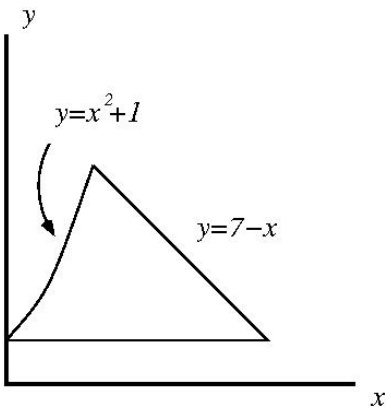
Math 221 Quiz #6 Solutions

1. Set up a *single* integral with respect to the variable y which calculates the area of the following region. Draw a sketch of the region. You do not need to evaluate the integral.

   The region bounded by the graphs of the functions $y = x^2 + 1$, $y = 1$ and $y = 7 - x$.

   Answer: The region in question can also be described as the region bounded by the graphs of the functions $x = \sqrt{y-1}$, $x = 7 - y$ and $y = 1$. An integral which calculates the area of this region is

$$\int_1^5 \left((7-y) - \sqrt{y-1} \right) \, dy.$$



2. Let $f(x) = e^x$ and consider $\int_0^2 f(x)\,dx$. For each of the following quantities, state whether you expect it to **overestimate**, **underestimate** or **exactly calculate** the value of this integral.

3. Any *left Riemann sum* approximation.

   Answer: Any left Riemann sum is an $\boxed{\text{underestimate}}$, since f is an increasing function on this interval.

4. Any *right Riemann sum* approximation.

   Answer: Any right Riemann sum is an $\boxed{\text{overestimate}}$, since f is an increasing function on this interval.

5. Any *Trapezoid Rule* approximation.

   Answer: Any Trapezoid Rule approximation is an $\boxed{\text{overestimate}}$, since f is a concave up function on this interval (the graph of f lies above any secant line).