

Math 221 Quiz #7 Solutions

1. Compute the volume of the object obtained by revolving the region between the graphs of the functions $y = x + 1$, $y = 2$ and $x = 0$ about the line $y = 1$.

Answer: The region in question is an equilateral right triangle with vertices $(0, 1)$, $(0, 2)$, $(1, 2)$. Revolving around the line $y = 1$ generates a solid of revolution whose volume is given by

$$\int_0^1 \pi ((2 - 1)^2 - (1 + x - 1)^2) dx = \pi \int_0^1 (1 - x^2) dx = \pi(1 - \frac{1}{3}) = \frac{2\pi}{3}.$$

2. Set up (but do **not** evaluate) an integral with respect to the variable x which computes the area of the region between the graphs of the functions $y = \frac{4}{x}$ and $y = 6 - 2x$. Then set up (but do **not** evaluate) another integral with respect to y which computes the same area.

Answer: Setting $\frac{4}{x} = 6 - 2x$ and solving for x gives $x = 1, 2$. The graphs of these two functions cross at $(1, 4)$ and $(2, 2)$ and the region in question lies within the rectangle with vertices $(1, 4)$, $(2, 4)$, $(1, 2)$, $(2, 2)$. Its area is equal to $\int_1^2 (6 - 2x - \frac{4}{x}) dx$ or $\int_2^4 (\frac{6-y}{2} - \frac{4}{y}) dy$.