Math 221 Recitation Exercises 5
Definition and interpretations of the derivative

A function $y = f(x)$ defined for x in an open interval $(x_0 - \delta, x_0 + \delta)$ is *differentiable at* x_0 if

$$\lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0} = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

exists. In this case, we call the value of this limit the *derivative of* f *at* x_0 and denote it by $f'(x_0)$. Other notations for the derivative include $\left.\frac{dy}{dx}\right|_{x=x_0}$, $\partial_x f(x_0)$ and $f_x(x_0)$.

The practical meaning of the derivative is:

"$f'(x_0)$ is the (instantaneous) rate of change of y with respect to x, when $x = x_0$."

The following examples are taken from section 2.4 of *Calculus: Single Variable*, second edition, by D. Hughes-Hallet, A. M. Gleason, et al, John Wiley and Sons, 1998.

Exercises

(1) The temperature T in degrees Fahrenheit of a dead body is given by $T = f(t)$, where t is the time in minutes after death. Assume that the initial temperature $f(0) = 98.6°$ F (body temperature) and that the temperature of the room is $75°$ F.

(a) What is the sign of $f'(t)$? Why?

(b) What are the units of $f'(20)$? What is the meaning of the statement $f'(20) = -1.5$?

(2) Let $g(t)$ be the number of centimeters of rain which has fallen since midnight, where t is the time in hours. Intepret each of the following equations, giving units in each case.

(a) $g(0) = 3.1$

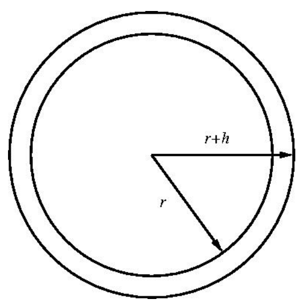
(b) $g'(8) = 0.4$

(c) $g^{-1}(10) = 16$

(3) The area of a circle of radius $r > 0$ is $A(r) = \pi r^2$.

(a) What is the practical meaning of the quantity $A'(5)$? $A'(r)$ for a general $r > 0$?

(b) Compute $A'(r) = \lim_{h\to 0} \frac{A(r+h)-A(r)}{h}$. Compare your answer to the formula $C(r) = 2\pi r$ for the circumference of such a circle. Use the following figure to explain why your answer is reasonable.



(4) The area of a square of side length $s > 0$ is $A(s) = s^2$ and the perimeter is $P(s) = 4s$. Show that

$$\lim_{h\to 0} \frac{A(s+2h) - A(s)}{h} = P(s).$$

Explain, drawing an appropriate figure (similar to the one in the previous problem).

Notice that $\lim_{h\to 0} \frac{A(s+2h)-A(s)}{h} = 2\lim_{h\to 0} \frac{A(s+2h)-A(s)}{2h} = 2A'(s)$.