

Math 285 Fall 2003

Course Overview

Topics covered:

1. First-order equations (Chapters 1 and 2 *except* 2.3 and 2.6)
 - a. analytic methods (separable, linear, exact equations, substitution methods)
 - b. graphical/qualitative methods (vector fields, phaselines)
 - c. numerical methods (Euler's method)

Applications: compound interest, population growth models, heating/cooling laws

Key words: *separable equation* (p. 30), *linear equation* (p. 44), *exact equation* (p. 66), *vector field* (p. 19), *autonomous equation* (p. 89), *critical/equilibrium point* (p. 89), *phaseline* (p. 89), *Euler's method* (p. 107).

2. Higher-order equations (Chapter 3, §§3.1–3.6 and 3.8)
 - a. analytic methods for unforced and forced constant coefficient equations
 - b. theoretical background (linear dependence/independence, Wronskians, existence and uniqueness theorems)

Applications: mechanical oscillations

Key words: *linear dependence/independence* (p. 158), *complementary vs. particular solutions* (p. 164), *Wronskian* (p. 159), *resonance* (p. 210), *eigenvalue/eigenfunction* (p. 230), *initial value problem vs. boundary value problem* (p. 228).

3. Fourier series techniques (Chapter 9)
 - a. analytic methods (computation of Fourier series, formal series solutions to ODE's and PDE's)
 - b. theoretical background (orthogonality of functions, convergence of Fourier series, rates of convergence vs. smoothness of functions)

Applications: evaluation of infinite sums ($\sum_{n=1}^{\infty} 1/n^2 = \pi^2/6$), mechanical oscillations

Key words: *periodicity* (p. 592), *orthogonality* (p. 593), *Fourier series/Fourier coefficient* (p. 595), *partial differential equation (PDE)* (p. 629).

Things which you should know how to do:

- solve differential equations of the various types studied in this course.
 - separable first-order
 - linear first-order
 - other first-order via substitution methods or method of exactness
 - linear constant coefficient of any order (forced or unforced), using Fourier series if necessary
 - heat, wave and Laplace equation
- use initial conditions to identify a particular solution.
- know how to generate and/or interpret a vector field or phase line (IODE project #1)
- use Euler's method to numerically approximate the solution of a first-order ODE.

You should be aware of the existence of more advanced methods (e.g., improved Euler) and some of their general properties (e.g., to what extent they give more accurate approximations), but you do not need to be able to carry out these methods in practice.
- understand the theoretical set-up for the solution of a forced linear equation (sum of complementary and particular solution)
- calculate the Wronskian and draw conclusions regarding linear dependence/independence
- use boundary conditions for an endpoint problem to find eigenvalues and eigenfunctions
- find the P -periodic extension of a function given on $[-L, L]$, $P = 2L$
- find the P -periodic odd or even extension of a function given on $[0, L]$, $P = 2L$.
- find the Fourier coefficients for such an extension
- sum infinite series by evaluating a Fourier series at an appropriate point
- understand and apply theoretical considerations for Fourier series (orthogonality of functions, rates of convergence vs. smoothness, as in IODE projects #4 and #5)
- solve ODE's or PDE's using Fourier series
- convert a verbal description into a mathematical model (differential equation plus initial/boundary conditions)