Homework #8 Solutions

3.2 #2 $\quad f(x) = 5 \quad g(x) = 2 - 3x^2 \quad h(x) = 10 + 15x^2$

linearly dependent on $\mathbb{R}$.

$$-4f(x) + 5g(x) + h(x) = 0$$

#6 $\quad f(x) = e^x \quad g(x) = \cosh(x) \quad h(x) = \sinh(x)$

linearly dependent on $\mathbb{R}$

$$-f(x) + g(x) + h(x) = 0$$

Note: $\cosh(x) = \frac{1}{2}(e^x + e^{-x}) \quad \sinh(x) = \frac{1}{2}(e^x - e^{-x})$

#15 $\quad y''' - 3y'' + 3y' - y = 0 \qquad y(0) = 2$
$y'(0) = 0$
$y''(0) = 0$

$$y(x) = C_1 e^x + C_2 x e^x + C_3 x^2 e^x$$

$$\begin{aligned} 2 &= y(0) = C_1 \\ 0 &= y'(0) = C_1 + C_2 \\ 0 &= y''(0) = C_1 + 2C_2 + 2C_3 \end{aligned}$$

$C_1 = 2 \qquad C_2 = -2 \qquad C_3 = 1$

$$\boxed{y(x) = 2e^x - 2xe^x + x^2 e^x}$$

#31 (a) $y'' + py' + qy = 0$

Evaluate at $x = a$.

$$y''(a) + py'(a) + qy(a) = 0$$

$$\Rightarrow \boxed{y''(a) = -py'(a) - qy(a)}$$

(b) $y'' - 2y' - 5y = 0$

$y(0) = 1$
$y'(0) = 0$
$y''(0) = C$

By a), any solution $y(x)$ to this problem satisfies

$$C = y''(0) = -(-2)(0) - (-5)(1) = 5.$$

3.3 #11 $y^{(4)} - 8y''' + 16y'' = 0$

$$r^4 - 8r^3 + 16r^2 = 0$$

$$r^2(r-4)^2 = 0 \qquad r = 0, 0, 4, 4$$

$$\boxed{y(x) = C_1 + C_2 x + C_3 e^{4x} + C_4 x e^{4x}}$$

#14. $y'''' + 3y'' - 4y = 0$

$$r^4 + 3r^2 - 4 = 0$$

$$(r^2 - 1)(r^2 + 4) = 0 \qquad r = 1, -1, 2i, -2i$$

$$\boxed{y(x) = C_1 e^{x} + C_2 e^{-x} + C_3 \cos(2x) + C_4 \sin(2x)}$$

#34. $3y''' - 2y'' + 12y' - 8y = 0$

$$y_1(x) = e^{2x/3}$$

$$3r^3 - 2r^2 + 12r - 8 = 0$$

We know: $r = \frac{2}{3}$ is a root. Factor out $r - \frac{2}{3}$:

$$\left(r - \tfrac{2}{3}\right)(3r^2 + 12) = 0$$

$$(3r - 2)(r^2 + 4) = 0$$

Other two roots are $r = \pm 2i$

$$\boxed{y(x) = C_1 e^{2x/3} + C_2 \cos(2x) + C_3 \sin(2x)}$$

#41. $y(x) = C_1 \cos(2x) + C_2 \sin(2x) +$

$C_3 \cosh(2x) + C_4 \sinh(2x)$

roots of the characteristic equation.

$r = \pm 2, \pm 2i$

$(r^2 - 4)(r^2 + 4) = 0$

$r^4 - 16 = 0$

$$\boxed{y^{(4)} - 16y = 0}$$