

Math 416 HW #3

due Wednesday, 2/13

1: Friedberg–Insel–Spence, §1.4, # 2 (b) (f).

2: (a) Let $\mathbf{x}_1, \dots, \mathbf{x}_n$ be a linearly independent collection of vectors in \mathbb{F}^n . Let $\mathbf{A} \in M_{n \times n}(\mathbb{F})$ be an invertible $(n \times n)$ matrix. Prove that the collection $\mathbf{A}\mathbf{x}_1, \dots, \mathbf{A}\mathbf{x}_n$ is linearly independent.

(b) Let $\mathbf{A} \in M_{n \times n}(\mathbb{F})$ satisfy the following condition:

Whenever $\mathbf{x}_1, \dots, \mathbf{x}_n$ is a linearly independent collection of vectors in \mathbb{F}^n , then $\mathbf{A}\mathbf{x}_1, \dots, \mathbf{A}\mathbf{x}_n$ is also a linearly independent collection.

Prove that \mathbf{A} is invertible.

3: Friedberg–Insel–Spence, §1.5, # 17.

4: Friedberg–Insel–Spence, §1.6, # 14.

5: Friedberg–Insel–Spence, §1.6, # 24.