Math 416 HW #6
due Friday, 4/4

1: Friedberg–Insel–Spence, §5.1 #11

2: Let $\mathbf{A} = (a_{ij})$ be an $n \times n$ matrix over $\mathbb{C}$ with characteristic polynomial $p_{\mathbf{A}}(\lambda)$. Prove that

$$p_{\mathbf{A}}(\lambda) = (a_{11} - \lambda)(a_{22} - \lambda) \cdots (a_{nn} - \lambda) + q(\lambda),$$

where $q(\lambda)$ is a polynomial of degree at most $n - 2$.

3: Friedberg–Insel–Spence, §5.2 #7. Also find an expression for $e^{t\mathbf{A}} = \exp(t\mathbf{A})$.

(Hint: the exponential of a matrix $\mathbf{A}$ is **defined** by the infinite series $\exp(\mathbf{A}) = I + \mathbf{A} + \frac{1}{2}\mathbf{A}^2 + \cdots + \frac{1}{n!}\mathbf{A}^n + \cdots$.)

4: Let $\mathbf{A}, \mathbf{B} \in M_{n\times n}(\mathbb{F})$ and assume that there exists an invertible matrix $\mathbf{P}$ so that $\mathbf{PAP}^{-1}$ and $\mathbf{PBP}^{-1}$ are **both** diagonal matrices. (In this case, we say that $\mathbf{A}$ and $\mathbf{B}$ are *simultaneously diagonalizable.*) Prove that if $\mathbf{A}$ and $\mathbf{B}$ are simultaneously diagonalizable, then $\mathbf{AB} = \mathbf{BA}$.

(Note: The converse assertion is also true but harder. We will prove it later in the course.)

5: Friedberg–Insel–Spence, §6.1 #15

6: Friedberg–Insel–Spence, §6.2 #11. More precisely, show that the rows of $\mathbf{A}$ form an orthonormal basis of $\mathbb{C}^n$ with respect to the standard inner product $\langle v, w\rangle = \sum_{j=1}^{n} v_j \overline{w_j}$, where $v = (v_1, \ldots, v_n)$ and $w = (w_1, \ldots, w_n)$ are in $\mathbb{C}^n$.