Math 441 Fall 2005
Quiz #1 Solutions

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1. Solve the initial value problem

$$y' + \frac{2}{x}y = 3, \qquad y(1) = 3.$$

Answer: This is a linear equation. The integrating factor is

$$I(x) = e^{\int^x \frac{2}{t}\,dt} = e^{2\ln|x|} = x^2.$$

Multiplying both sides by x^2 gives

$$(x^2y)' = x^2y' + 2xy = 3x^2$$

so

$$x^2y = x^3 + C.$$

Using the initial condition $y(1) = 3$ we get $3 = 1 + C$ or $C = 2$, so

$\boxed{y(x) = \frac{x^3+2}{x^2}}$

2. Find the general solution to the differential equation

$$(2x\ln|y| + y^2) + (\frac{x^2}{y} + 2xy)\,\frac{dy}{dx} = 0.$$

Answer: Let $M(x,y) = 2x\ln|y| + y^2$ and $N(x,y) = \frac{x^2}{y} + 2xy$. Since

$$M_y = \frac{2x}{y} + 2y = N_x$$

the equation is exact. The solution is $\psi(x,y) = C$, where $\psi_x = M$ and $\psi_y = N$. We compute

$$\psi(x,y) = \int^x M(t,y)\,dt = x^2\ln|y| + xy^2 + C_1(y)$$

and

$$\psi(x,y) = \int^y N(x,t)\,dt = x^2\ln|y| + xy^2 + C_2(x).$$

Comparing these solutions, we see that $C_1(y) = 0 = C_2(x)$. The implicit solution to the equation is

$\boxed{x^2\ln|y| + xy^2 = C}$