

## Math 441 Fall 2005 Quiz #5 Solutions

1. (10 points) Consider the second order ODE

$$y'' + 4y' + 5y = e^t + t^2 + e^{-2t} \cos t.$$

Determine an appropriate guess for a particular solution  $y_p$  by the method of undetermined coefficients. Do **not** attempt to solve for the coefficients in your guess!

*Answer:* The complementary equation  $y'' + 4y' + 5y = 0$  has general solution  $y_c(t) = C_1 e^{-2t} \cos t + C_2 e^{-2t} \sin t$ . Thus a suitable guess for the method of undetermined coefficients is

$$y_p(x) = Ae^t + Bt^2 + Ct + D + Ete^{-2t} \cos t + Fte^{-2t} \sin t.$$

2. Solve the initial value problem  $y'' + y = \sec^3(x)$ ,  $y(0) = 1$ ,  $y'(0) = 2$ .

*Answer:* The complementary equation  $y'' + y = 0$  has general solution  $y_c(x) = C_1 \cos x + C_2 \sin x$ . We guess

$$y_p(x) = u_1(x) \cos x + u_2(x) \sin x.$$

Then

$$y'_p(x) = [u_1(x)(-\sin x) + u_2(x) \cos x] + [u'_1(x) \cos x + u'_2(x) \sin x].$$

We require

$$u'_1 \cos x + u'_2 \sin x = 0$$

and compute

$$y''_p(x) = [u_1(x)(-\cos x) + u_2(x)(-\sin x)] + [u'_1(x)(-\sin x) + u'_2(x) \cos x].$$

Thus

$$y''_p(x) + y_p(x) = u'_1(x)(-\sin x) + u'_2(x) \cos x = \sec^3 x.$$

The solution to this system of equations in  $u'_1$  and  $u'_2$  is

$$u'_1(x) = -\sec^2 x \tan x \quad \text{and} \quad u'_2(x) = \sec^2 x.$$

Integrating gives

$$u_1(x) = -\frac{1}{2} \sec^2 x \quad \text{and} \quad u_2(x) = \tan x$$

so

$$y_p(x) = -\frac{1}{2} \sec^2 x \cos x + \tan x \sin x = \frac{\sin^2 x}{\cos x} - \frac{1}{2 \cos x} = \frac{1}{2} \sec x - \cos x$$

and

$$y(x) = C_1 \cos x + C_2 \sin x + \frac{1}{2} \sec x - \cos x.$$

The term  $-\cos x$  can be absorbed into the term  $C_1 \cos x$ , giving

$$y(x) = C_1 \cos x + C_2 \sin x + \frac{1}{2} \sec x.$$

The initial conditions give  $1 = y(0) = C_1 + \frac{1}{2}$  and  $2 = y'(0) = C_2$ , so

$$y(x) = \frac{1}{2} \cos x + 2 \sin x + \frac{1}{2} \sec x.$$