Math 490 Homework #3
due Monday, 9/24

1: Kinsey, Exercise 3.21. (We discussed this fact briefly in class. Write out the complete argument carefully.)

2: Let $A_1, A_2, A_3, \ldots$ be a sequence of nonempty compact subsets of $\mathbb{R}^n$. Assume that $A_1 \supset A_2 \supset A_3 \supset \cdots$. Prove that $\cap_{n=1}^{\infty} A_n$ is nonempty.

(Start of the proof: Suppose $\cap_{n=1}^{\infty} A_n$ is empty. By the Heine–Borel theorem, $U_n = \mathbb{R}^n \setminus A_n$ is open for each n. Verify that the sets $U_2, U_3, U_4, \cdots$ are an open cover of the compact set A_1.)

3: Let (X, T) be a Hausdorff topological space. Prove that $\{x\}$ is a closed set for each $x \in X$.

4: Let (X, T) be any compact topological space and let $Y \subset \mathbb{R}^n$ for some n. Let $f : X \to Y$ be a continuous, one-to-one, and onto function. Prove that f is a homeomorphism.

(Hint: it suffices to prove that f maps closed subsets of X to closed subsets of Y. Why?)

5: Kinsey, Exercise 4.6.