

Math 490 Homework #5

due Friday, 10/12

1: (Minimal triangulations of simplicial surfaces) For any simplicial 2-complex K , let V, E, F be the cardinalities of K^0, K^1, K^2 , respectively. Assume that $|K|$ is homeomorphic with a compact surface S , and let $\chi = \chi(S)$ be the Euler characteristic of S .

(a) Prove that $3F = 2E$, $E = 3(V - \chi)$, and

$$V \geq \frac{7 + \sqrt{49 - 24\chi}}{2}.$$

(Hint: find an inequality relating E and V .)

(b) Find lower bounds for V, E, F in each of the following cases: (i) $S = S^2$, (ii) $S = T^2$, (iii) $S = P^2$, (iv) $S = K^2$.

(c) Find triangulations of S^2, T^2 and P^2 which achieve the minimal values for V, E, F in part (b).

(d) There is no triangulation of the Klein bottle K^2 which achieves the minimal values for V, E, F in part (b)(iv). (You do not need to prove this.) Find a triangulation of K^2 using $V_0 + 1$ vertices, where V_0 is the lower bound which you found in (b)(iv).

2: Kinsey, Exercise 5.21.

3: Kinsey, Exercise 5.19.

A *semi-regular* (or *Archimedean*) polyhedron is a finite polyhedron P in \mathbb{R}^3 with the following property: the faces of P are all regular polygons,¹ and at each vertex of P , the same number and type of polygons meet. An example is the *truncated icosahedron*,² whose faces are all either regular pentagons or regular hexagons, with two hexagons and one pentagon meeting at every vertex. This Archimedean solid is made up of twelve pentagons and twenty hexagons, for a total of 32 faces, and has 90 edges and 60 vertices.

4: Classify all of the Archimedean (including Platonic) polyhedra which are comprised of at most two types of polygons (e.g., m -gons and n -gons, $m, n \geq 3$) with the property that four faces meet at every vertex.

¹not necessarily all with an equal number of edges!

²or soccer ball. This structure has also become famous as the chemical structure of the *buckyball* C_{60} .