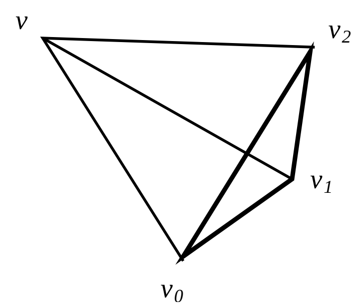
Math 490 Homework #6
due Friday, 11/9

1: Kinsey, Exercises 6.2 and 6.6.

2: Let $\sigma =< v_0, v_1, \ldots, v_k >$ be a k-simplex in $\mathbb{R}^n$, $k < n$. Let $v \in \mathbb{R}^n$ be another point so that the set $\{v, v_0, v_1, \ldots, v_k\}$ is affinely independent. The *cone over* σ *with apex* v is the $(k+1)$-simplex

$$v \cdot \sigma =< v, v_0, v_1, \ldots, v_k > .$$



(a) Prove: $\partial(v \cdot \sigma) = \sigma - v \cdot \partial\sigma$.

Now let K be a simplicial k-complex, and let v be another point so that the set $\{v\} \cup K^0$ is affinely independent. The *cone over* K *with apex* v is the simplicial $(k+1)$-complex

$$v \cdot K$$

which has $(v \cdot K)^0 = \{v\} \cup K^0$ and

$$(v \cdot K)^{i+1} = \{v \cdot \sigma : \sigma \in K^i\} \qquad i = 0, 1, 2, \ldots, k.$$

(b) Let K be a compact connected simplicial 1-dimensional manifold. (Note: every such manifold is homeomorphic to S^1.) Draw a picture of $v \cdot K$. If K has n vertices and n edges, compute the number of i-simplices ($i = 0, 1, 2$) V, E, F for $v \cdot K$ as well as $\chi(v \cdot K)$ (your answers may depend on n). What space is $v \cdot K$ up to homeomorphism?

(c) Now let K be **any** simplicial 1-complex (not necessarily representing a manifold), with V_0 vertices and E_0 edges. Compute V, E, F for $v \cdot K$ as well as $\chi(v \cdot K)$ in terms of V_0 and E_0.

(d) **Extra credit** Let K be any simplicial 1-complex. Calculate the homology groups of the simplicial 2-complex $v \cdot K$.

See Kinsey, pp. 138–139 for more details on these notions.

3: Kinsey, Exercise 6.22.

4: Kinsey, Exercise 7.4.

5: Kinsey, Exercise 7.5.

6: Kinsey, Exercise 7.14.