
2: Let $\sigma = \langle v_0, v_1, \ldots, v_k \rangle$ be a $k$-simplex in $\mathbb{R}^n$, $k < n$. Let $v \in \mathbb{R}^n$ be another point so that the set $\{v, v_0, v_1, \ldots, v_k\}$ is affinely independent. The cone over $\sigma$ with apex $v$ is the $(k + 1)$-simplex

$$v \cdot \sigma = \langle v, v_0, v_1, \ldots, v_k \rangle.$$

(a) Prove: $\partial(v \cdot \sigma) = \sigma - v \cdot \partial \sigma$.

Now let $K$ be a simplicial $k$-complex, and let $v$ be another point so that the set $\{v\} \cup K^0$ is affinely independent. The cone over $K$ with apex $v$ is the simplicial $(k + 1)$-complex

$$v \cdot K$$

which has $(v \cdot K)^0 = \{v\} \cup K^0$ and

$$(v \cdot K)^{i+1} = \{v \cdot \sigma : \sigma \in K^i\} \quad i = 0, 1, 2, \ldots, k.$$

(b) Let $K$ be a compact connected simplicial 1-dimensional manifold. (Note: every such manifold is homeomorphic to $S^1$.) Draw a picture of $v \cdot K$. If $K$ has $n$ vertices and $n$ edges, compute the number of $i$-simplices ($i = 0, 1, 2$) $V, E, F$ for $v \cdot K$ as well as $\chi(v \cdot K)$ (your answers may depend on $n$). What space is $v \cdot K$ up to homeomorphism?

(c) Now let $K$ be any simplicial 1-complex (not necessarily representing a manifold), with $V_0$ vertices and $E_0$ edges. Compute $V, E, F$ for $v \cdot K$ as well as $\chi(v \cdot K)$ in terms of $V_0$ and $E_0$.

(d) Extra credit Let $K$ be any simplicial 1-complex. Calculate the homology groups of the simplicial 2-complex $v \cdot K$.

See Kinsey, pp. 138–139 for more details on these notions.


5: Kinsey, Exercise 7.5.