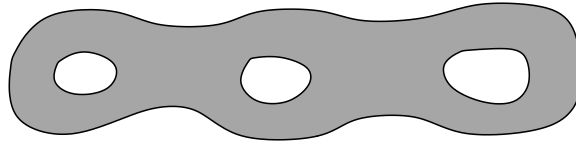


Math 490 Homework #7

due Friday, 11/16

- 1: In class we proved the Brouwer Fixed Point Theorem for the closed unit disc $D = \overline{B(0,1)} = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\}$: every continuous function $f : D \rightarrow D$ has a fixed point. Equivalently, D has the fixed point property (see Kinsey, Definition 2.31).

For each $n \in \mathbb{N} \cup \{0\}$, let A_n be the n -holed annulus, i.e., A_n is the set (unique up to homeomorphism) obtained by removing n open discs with disjoint closures from D . Thus $A_0 = D$, A_1 is an annulus (ring) in \mathbb{R}^2 , and so on. The following figure illustrates A_3 .



- (a) Show that A_1 does not have the fixed point property.
- (b) Show that A_3, A_4, A_5, \dots do not have the fixed point property.
- (c) (**Extra credit**) Does A_2 have the fixed point property?
- 2: Consider the planar vector field $V(x,y) = (y, 1 - x^2)$.
- (a) Find the critical points and classify them according to type and stability. What are the indexes of these critical points?
- (b) Compute the winding number of V along the following curves:
- (i) $\{(x,y) : x^2 + y^2 = 2x\}$
- (ii) $\{(x,y) : x^2 + y^2 = -2x\}$
- (iii) $\{(x,y) : x^2 + y^2 = 2y\}$
- (iv) $\{(x,y) : x^2 + y^2 = 4\}$
- Verify the Poincaré Index theorem in each case.
- 3: Prove: if P is an isolated critical point of a planar vector field V which is not a center or a focus, then the sum of the number of elliptic and hyperbolic sectors of V at P is even.

4: The following diagram is the phase portrait of a vector field on a certain compact connected surface. State the topological type and Euler characteristic of the surface, classify all of the critical points by type, and compute the index of each critical point. Verify the Poincaré Index Theorem.

5: Kinsey, Exercise 11.23.

6: Kinsey, Exercise 11.24.