Math 490 Homework #8
due Friday, 11/30

A *torus knot/link* is a knot or link which can be embedded in the standard torus T^2. A basic example is the (q, r)*-torus knot (link)* $K(q, r)$, where q and r are positive integers.

It is easy to check that $K(q, 1)$ and $K(1, r)$ are equivalent with the unknot, so assume $q, r \geq 2$. Fact (you do not need to prove this): $K(q, r)$ and $K(q', r')$ are equivalent iff $\{q, r\} = \{q', r'\}$.

1: Show that $K(3, 2)$ is equivalent with the trefoil knot.

2: (a) Let g be the greatest common divisor of q and r. Show that $K(q, r)$ is a link with g components. (Thus $K(q, r)$ is a knot if and only if q and r are relatively prime.)

(b) By part (a), $K(4, 2)$ is a 2-component link. Calculate its linking number.

3: Let L be a 2-component link. Show that the linking number of the components of L is an integer.

4: Classify the Seifert surface S_L for the link L consisting of a pair of unlinked unknots (shown in the following diagram). What is the genus of S_L? How many boundary components does it have? How does this surface differ from the Seifert surface for the Hopf link (discussed in class)?