

Math 542 HW #2

due Friday, 9/7

- 1: #III.6.9 from Palka.
- 2: #III.6.57(i)(iv) from Palka.
- 3: #III.6.62 from Palka
- 4: #III.6.64 from Palka.

Define

$$H(z_0) := \frac{\max_{-\pi < \theta \leq \pi} |\partial_\theta f(z_0)|}{\min_{-\pi < \theta \leq \pi} |\partial_\theta f(z_0)|}.$$

Note that $H(z_0) = 1$ if f is analytic at z_0 ; the quantity $H(z_0)$ is a measure of the deviation from conformality of f at z_0 .

- 5: #III.6.56 from Palka. Calculate $H(z_0)$ for each $z_0 \in \mathbb{C} \setminus \{0\}$ and show that

$$\sup_{z_0 \in \mathbb{C} \setminus \{0\}} H(z_0) = \max\left\{\alpha, \frac{1}{\alpha}\right\}.$$

- 6: Find the general form of a rational function f which satisfies $|f(z)| = 1$ whenever $|z| = 1$. Describe how the zeros and poles of f are related to each other.