Math 542 HW #8
due Monday, 10/29

1: #VII.5.88 from Palka. Show by an example that the converse can fail: a family $\mathcal{F}$ of analytic functions in a domain $\Omega \subset \mathbb{C}$ with the property that $\{f' : f \in \mathcal{F}\}$ is normal, need not itself be a normal family. Can you add an extra hypothesis sufficient to guarantee that $\mathcal{F}$ is normal?

2: #VII.5.90 from Palka.

3: #VII.5.91 from Palka.

4: #VII.5.96 from Palka.

5: Denote by $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$ the open unit disc. Fix $M < \infty$. Let $\mathcal{F}$ be the family of all analytic functions $f : \mathbb{D} \to \mathbb{C}$ with the property that

$$\iint_{\mathbb{D}} |f(z)|^2(1 - |z|^2)\, dA(z) \le M,$$

where $dA(z) = dx\, dy$ denotes the area measure in the plane. Show that $\mathcal{F}$ is a normal family.