

Math 230 Section BE1/BB1 Review for Midterm Exam III

The third midterm exam will take place on **Monday, April 4** in class. It will cover Chapters 8, 9 and 10: techniques of integration (integration by parts, partial fractions, trig integrals and trig substitutions), improper integrals, and Taylor and MacLaurin approximations to functions. Use of a graphing calculator (through TI-86) will be allowed.

You should be able to:

- use a variety of techniques to compute definite or indefinite integrals.
 - Integration by parts: This was covered on the last exam, so it will not be tested on its own on this exam. However, it may be necessary to integrate by parts within the context of a more general problem.
 - Partial Fractions: State the general form of the partial fraction decomposition (with undetermined coefficients) for a particular rational function. Use long division if necessary to reduce an improper fraction to the sum of a polynomial and a proper fraction. Find the undetermined coefficients (by the method of residues, by setting up a system of linear equations in the coefficients and solving, or by a combination of these methods).
 - Trig substitutions: Evaluate integrals of products and powers of trig functions, using reduction formulas (integration by parts), double angle formulas, etc. Evaluate integrals involving expressions of the form

$$\sqrt{a^2 - x^2}, \quad \sqrt{x^2 - a^2}, \quad \sqrt{x^2 + a^2}$$

by making a trig substitution for x .

(Suggestion: for integrals involving $\sqrt{a^2 - x^2}$, substitute $x = a \sin \theta$; for integrals involving $\sqrt{x^2 - a^2}$, substitute $x = a \sec \theta$; and for integrals involving $\sqrt{x^2 + a^2}$, substitute $x = a \tan \theta$. This avoids introducing some extra negative signs into the integrand.)

- combine these techniques as necessary; diagnose a given integral (i.e., decide which technique is most appropriate). Some integrals can be evaluated by more than one method, however, it's often the case that one method is significantly simpler than the others. You may need to try more than one method; do your preliminary work on scratch paper and only write the final derivation on the test!
- evaluate improper integrals. Be familiar with the definition of an improper integral as a limit of proper integrals. Understand the difference between *convergent* and *divergent* integrals and how to distinguish between them. Use comparison tests such as the p -tests:

$$\int_0^1 \frac{dx}{x^p}$$

converges if $p < 1$ and diverges if $p \geq 1$, while

$$\int_1^\infty \frac{dx}{x^p}$$

converges if $p > 1$ and diverges if $p \leq 1$. Estimate the value of an improper integral by “bounding the tail”.

- compute Taylor and MacLaurin approximations to a function. The n th Taylor approximation to a function $y = f(x)$ at $x = x_0$ is

$$P_n(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{1}{2}f''(x_0)(x - x_0)^2 + \cdots + f^{(n)}(x_0)(x - x_0)^n.$$

MacLaurin approximations are the special case $x_0 = 0$. Use the error bound theorem to give accuracy guarantees for Taylor and MacLaurin approximations. If

$$|f^{(n+1)}(x)| \leq K$$

for all x in an interval $[x_0 - \epsilon, x_0 + \epsilon]$, then

$$|f(x) - P_n(x)| \leq \frac{K|x - x_0|^{n+1}}{(n+1)!} \leq \frac{K\epsilon^{n+1}}{(n+1)!}$$

for all x in $[x_0 - \epsilon, x_0 + \epsilon]$.

Some practice problems

1. Evaluate $\int \frac{15x+9}{x(x^2-9)} dx$.
2. Evaluate $\int \frac{x^2}{(1+x^2)^2} dx$ using a trig substitution.
3. For which values of p does

$$\int_2^{\infty} \frac{dx}{x(\ln x)^p}$$

converge? For which values of p does it diverge? When it converges, what value does it converge to?

4. (a) Does $\int_0^4 \frac{dx}{\sqrt{4-\sqrt{x}}}$ converge or diverge?
(b) Does $\int_0^4 \frac{dx}{\sqrt{4-x}}$ converge or diverge?
5. Find the fourth MacLaurin approximation to $f(x) = \ln(1+x)$.
6. Find the third MacLaurin approximation to $f(x) = \frac{x}{1+x^2}$.

Solutions

1. $3 \ln|x-3| - 2 \ln|x+3| - \ln|x| + C$
2. $\frac{1}{2} \arctan x - \frac{x}{2(1+x^2)} + C$
3. converges for $p > 1$ to $\frac{1}{(p-1)(\ln 2)^{p-1}}$; diverges for $p \leq 1$
4. (a) diverges; (b) converges to the value 6
5. $P_4(x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4$
6. $P_3(x) = x - x^3$