Math 230 Section BE1/BB1
Review for the Final Exam

Date: **Friday, May 13**

Time: **8:00am** to **11:00am**

Place: **140 Henry Building** (regular classroom)

Topics: Everything in Chapters 5–11 and Chapter V except sections 7.5, 9.3, 10.3; also handouts on Simpson's Rule, surface area, differential equations, parametric equations.

You should be able to:

- Use a variety of techniques to compute definite/indefinite integrals: substitution, partial fractions, integration by parts, trig substitutions and identities.

  Reference: Chapters 5 and 8

- Approximate integrals numerically using Riemann sums, Simpson's rule, etc. Use the Error Bound Theorem to draw conclusions about the accuracy of such approximations.

  Reference: Chapter 6

- Interpret definite integrals geometrically and physically. Set up and evaluate integrals to compute arc length, volume, surface area, work.

  Reference: Chapter 7

- Analyze a differential equation. Solve separable differential equations by the method of separation of variables. Interpret a slope field (vector field) for a first-order differential equation. Use Euler's method to generate approximate solutions to differential equations.

  Reference: sections 2.5, 4.1, 6.3, and 7.4

- Set up/evaluate improper integrals. Distinguish convergent vs. divergent integrals. Estimate the value of a convergent improper integral by "bounding the tail".

  Reference: Chapter 10

- Analyze sequences and series. Test an infinite series for convergence/divergence (using the Ratio Test, Alternating Series Test, Integral Test, etc.). Estimate the value of a convergent infinite series by "bounding the tail". Find the interval/radius of convergence for a power series. Calculate the Taylor/Maclaurin series and/or the Taylor/Maclaurin polynomial approximations for a function.

  Reference: Chapters 9 anad 11

- Work with functions in parametric or polar form. Graph systems of parametric equations and polar curves. Recognize conic sections in parametric or polar form. Compute arc length, slope, area, ... in polar coordinates.

  Reference: section 4.4 and Chapter V

- **Integration by parts:** $\int u\,dv = uv - \int v\,du$
- **Riemann sums** for $I = \int_a^b f(x)\,dx$:
  - $L_n = \sum_{i=0}^{n-1} f(x_i)\,\Delta x$ (left Riemann sum with n subdivisions),
  - $R_n = \sum_{i=1}^{n} f(x_i)\,\Delta x$ (right Riemann sum with n subdivisions),
  - $T_n = (L_n + R_n)/2$ (trapezoid sum with n subdivisions),
  - $M_n = \sum_{i=1}^{n} f(\frac{x_{i-1}+x_i}{2})\,\Delta x$ (midpoint sum with n subdivisions),
  - $S_{2n} = (2M_n + T_n)/3$ (Simpson's rule with $2n$ subdivisions).
- **Error bound theorems for approximating sums**: Let K_n be an upper bound for $|f^{(n)}(x)|$ on the interval $[a, b]$. Then

$$|I - L_n|, |I - R_n| \le \frac{K_1(b-a)^2}{2n},$$

$$|I - T_n| \le \frac{K_2(b-a)^3}{12n^2}, \quad |I - M_n| \le \frac{K_2(b-a)^3}{24n^2},$$

$$|I - S_{2n}| \le \frac{K_4(b-a)^5}{180n^4}.$$

- **Definite integrals in geometry:**

  arc length of graph of $y = f(x)$ from $x = a$ to $x = b$ is $\int_a^b \sqrt{1 + f'(x)^2}\,dx$

  volume of solid S obtained by revolving $y = f(x)$ from $x = a$ to $x = b$ about the x-axis is $\pi \int_a^b f(x)^2\,dx$

  surface area of S is $2\pi \int_a^b f(x)\sqrt{1 + f'(x)^2}\,dx$
- **p-test for integrals:** $\int_1^\infty \frac{dx}{x^p}$ converges if $p > 1$ and diverges if $p \le 1$; $\int_0^1 \frac{dx}{x^p}$ converges if $p < 1$ and diverges if $p \ge 1$.
- **convergence tests for series:**
  - **Integral test:** if $a(x)$ is continuous, positive and decreasing, then $\sum_{k=1}^\infty a(k)$ converges if and only if $\int_1^\infty a(x)\,dx$ converges.
  - **geometric series:** $\sum_{n=1}^\infty ar^n$ converges if $|r| < 1$ (to $\frac{a}{1-r}$) and diverges if $|r| \ge 1$.
  - **p-test for series:** $\sum_{n=1}^\infty \frac{1}{n^p}$ converges if $p > 1$ and diverges if $p \le 1$.
  - **Ratio test:** Let $\sum_{k=1}^\infty a_k$ be an infinite series for which $L = \lim_{k\to\infty} \left|\frac{a_{k+1}}{a_k}\right|$ exists. If $L < 1$ then the series converges; if $L > 1$ then the series diverges; if $L = 1$ the series may converge or diverge (the test is inconclusive).
  - **Alternating series test:** Let $\sum_{k=1}^\infty (-1)^{k+1} a_k$ be an infinite alternating series ($a_k > 0$) whose terms a_k are decreasing with $\lim_{k\to\infty} a_k = 0$. Then the series converges.
- The **Taylor series** for an infinitely diff'ble function $f(x)$ at $x = x_0$ is $\sum_{n=0}^\infty \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n$.
- The n**th Taylor approximation** to $f(x)$ at $x = x_0$ is $P_n(x) = \sum_{k=0}^n \frac{f^{(k)}(x_0)}{k!}(x - x_0)^k$.
- **Error bound theorems:** if K_{n+1} is an upper bound for $|f^{(n+1)}(x)|$ on $[x_0 - \epsilon, x_0 + \epsilon]$ then

$$|f(x) - P_n(x)| \le \frac{K_{n+1}|x - x_0|^{n+1}}{(n+1)!}$$

  for all x with $x_0 - \epsilon \le x \le x_0 + \epsilon$.
- **Parametric equations:** $x = f(t)$, $y = g(t)$.
- Arc length of the graph of this system from $t = a$ to $t = b$ is $\int_a^b \sqrt{f'(t)^2 + g'(t)^2}\,dt$.
- Slope of the graph at $t = t_0$ is $\frac{dy}{dx} = \frac{g'(t_0)}{f'(t_0)}$.
- **Polar coord's:** $x = r\cos\theta$, $y = r\sin\theta$.
- **Conic sections** (ellipses, parabolas, hyperbolas) in polar coord's are given by $r = \frac{pe}{1 \pm e\cos\theta}$, where p, e are parameters.
- **Arc length** of a polar curve $r = F(\theta)$ from $\theta = a$ to $\theta = b$ is $\int_a^b \sqrt{F(\theta)^2 + F'(\theta)^2}\,d\theta$.
- Slope of $r = F(\theta)$ at $\theta = \theta_0$ is

$$\frac{dy}{dx} = \frac{F'(\theta)\sin\theta + F(\theta)\cos\theta}{F'(\theta)\cos\theta - F(\theta)\sin\theta}.$$

- Area of region bounded by $r = F(\theta)$, $\theta = a$, and $\theta = b$ is $\frac{1}{2}\int_a^b F(\theta)^2\,d\theta$.