

# Math 385 Spring 2007

## First order linear ordinary differential equations

A differential equation is called **first order linear** if it can be put in the form

$$\frac{dy}{dx} + P(x)y = Q(x) \tag{1}$$

for some functions  $P$  and  $Q$ .

### Method for solving first order linear equations.

Step 0. Put the equation in the standard form (1).

Step 1. Find the antiderivative  $\int P(x) dx$  of  $P$ .

**NOTE:** You don't need to include a constant of integration in this step.

Step 2. Exponentiate the result of Step 1 to obtain  $e^{\int P(x) dx}$ .

Step 3. Multiply both sides of (1) by  $e^{\int P(x) dx}$ .

Step 4. Use the "reverse product rule" to simplify the left hand side of the equation.

Step 5. Antidifferentiate both sides. (This time you do need to include a constant of integration  $C$ .)

Step 6. Solve for  $y$ . Use the initial condition (if there is one) to determine  $C$ .

**Example 1.** Solve  $\frac{dT}{dt} = -k(T - A)$ .

**Example 2.** Solve  $y' + 2xy = e^{-x^2}$ .

**Example 3.** Solve

$$\frac{dx}{dt} + cx = a \cos(kt) + b \sin(kt).$$