Solutions

Homework 1 Math 461 Chapter 1 #4,5,7,8,9,11,14,18,19,20

#4 There are $4! = 24$ arrangements if each of the boys can play any instrument.

If Jay and Jack can only play piano and drums, there are 2 ways to assign instruments to Jay and Jack, which leaves two possible assignments for John and Jim. Thus there are $2 \cdot 2 = 4$ possible arrangements.

#5 There are 8 choices for the first digit (any number between 2 and 9) 2 choices for the second (0 and 1) and 9 choices for the third digit. By the counting principle we have
$8 \cdot 2 \cdot 9 = 144$ choices.

Number of area codes starting with a 4: $1 \cdot 2 \cdot 9 = 18$ choices

#7 a) There are six children altogether and thus there are $6!$ ways they can sit in a row (Permutations of 6 objects)

b) $3!$ to arrange the girls and $3!$ to arrange the boys. But the boys can sit on the left (thus the girls on the right) and vice versa. Thus
$3!\,3!\,2 = 72$ arrangements

c) Now we have a block of boys and 3 girls. (4 objects) There are $4!$ ways to arrange the block of boys and the 3 girls in a row and there $3!$ ways to arrange the boys within the block. By the counting principle we have $4!\,3! = 144$ different arrangements.

d) Either the girls are in the even seats (and the boys are in the odd seats) or vice versa. Thus there are
$2 \cdot 3!\,3! = 72$ arrangements

#8 a) $5!$

b) $\frac{7!}{2!2!}$ (seven letters including two Ps and two Os)

c) $\frac{11!}{4!4!2!}$ (eleven letters, 4 Is, 4 Ss and two Ps)

d) $\frac{7!}{2!2!}$ (seven letters, two As, two Rs)

#9 12! possible permutations but the 6 black blocks are indistinguishable and the same holds for the 4 red. Thus

$$\frac{12!}{6!4!1!1!} = 27,720 \text{ arrangements}$$

#11 This is similar to problem 7.

a) $6! = 720$

b) $3!$ ways for the , $2!$ for the math books and $1!$ for the chemistry. But there are $3!$ permutations of the 3 groups according to subject. Thus $3!2!3! = 72$ different arrangements.

c) $4!$ arrangements for the 4 distinct groups (3 novels, 1 math, 1 math, 1 chemistry) and 3! permutations of the 3 novels. Thus $4!3! = 144$ arrangements.

#14 $\binom{52}{5} = \frac{52!}{47!5!} = \frac{48 \cdot 49 \cdot 50 \cdot 51 \cdot 52}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} = \cdot 49 \cdot 10 \cdot 17 \cdot 52$

$= 2,598,960$

#18 $\binom{5}{2}$ ways to pick the Republicans, $\binom{6}{2}$ ways to pick the Democrats and $\binom{4}{3}$ ways to pick the Independents. By the counting principle there are

$\binom{5}{2} \cdot \binom{6}{2} \cdot \binom{4}{3} = 600$ ways to form a committee.

#19 a) If we pick the two men that are feuding then we can fill in the last spot in 4 different ways since any of the remaining 4 men will do. So we have 4 bad choices.

Thus the number of good choices for men are $\binom{6}{3} - 4$. We also have $\binom{8}{3}$ choices for the women and thus

$\binom{8}{3} \cdot \left[\binom{6}{3} - 4\right] = 896$ good choices

b) Similarly $\binom{6}{3}\left[\binom{8}{3} - 6\right] = 1000$ good choices

c) We pick the man and woman that are feuding and

and then we have $\binom{5}{2}$ ways to pick 2 men from the remaining 5

and $\binom{7}{2}$ ways to pick 2 women from the remaining 7.

Thus the bad choices are $\binom{5}{2}\binom{7}{2}$.

Thus $\binom{8}{3}\binom{6}{3} - \binom{7}{2}\binom{5}{2} = 910$ good choices

#20 a) Pick the 2 feuding friends and then choose the next

3 from the remaing 6. Thus $\binom{6}{3}$ bad choices.

Thus $\binom{8}{5} - \binom{6}{3} = 36$ good choices left.

b) Let's invite the 2 friends. Then we fill in the 3 spots from 6 people and we have $\binom{6}{3}$ choices.

If we don't invite the 2 friends then we choose 5 from the remaining 6. Thus $\binom{6}{5}$ choices.

In total we have $\binom{6}{3} + \binom{6}{5} = 26$ possibilities